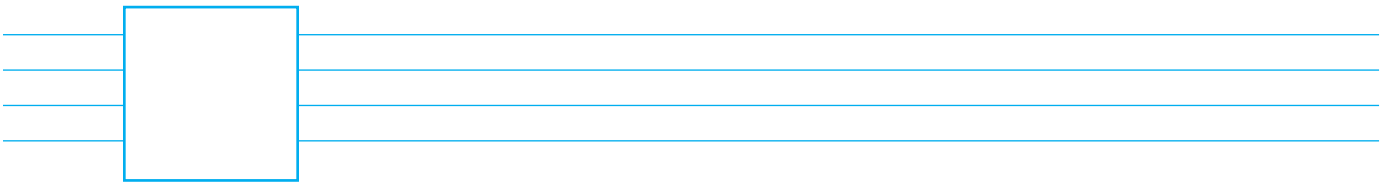


CALCULUS

EARLY TRANSCENDENTALS

SIXTH EDITION

JAMES STEWART

McMASTER UNIVERSITY

THOMSON

BROOKS/COLE

AUSTRALIA ▪ BRAZIL ▪ CANADA ▪ MEXICO ▪ SINGAPORE ▪ SPAIN ▪ UNITED KINGDOM ▪ UNITED STATES

DIAGNOSTIC TESTS

Success in calculus depends to a large extent on knowledge of the mathematics that precedes calculus: algebra, analytic geometry, functions, and trigonometry. The following tests are intended to diagnose weaknesses that you might have in these areas. After taking each test you can check your answers against the given answers and, if necessary, refresh your skills by referring to the review materials that are provided.

A DIAGNOSTIC TEST: ALGEBRA

1. Evaluate each expression without using a calculator.

(a) $(-3)^4$ (b) -3^4 (c) 3^{-4}

(d) $\dfrac{5^{23}}{5^{21}}$ (e) $\left(\dfrac{2}{3}\right)^{-2}$ (f) $16^{-3/4}$

2. Simplify each expression. Write your answer without negative exponents.

(a) $\sqrt{200} - \sqrt{32}$

(b) $(3a^3b^3)(4ab^2)^2$

(c) $\left(\dfrac{3x^{3/2}y^3}{x^2y^{-1/2}}\right)^{-2}$

3. Expand and simplfy.

(a) $3(x + 6) + 4(2x - 5)$ (b) $(x + 3)(4x - 5)$

(c) $\left(\sqrt{a} + \sqrt{b}\,\right)\left(\sqrt{a} - \sqrt{b}\,\right)$ (d) $(2x + 3)^2$

(e) $(x + 2)^3$

4. Factor each expression.

(a) $4x^2 - 25$ (b) $2x^2 + 5x - 12$

(c) $x^3 - 3x^2 - 4x + 12$ (d) $x^4 + 27x$

(e) $3x^{3/2} - 9x^{1/2} + 6x^{-1/2}$ (f) $x^3y - 4xy$

5. Simplify the rational expression.

(a) $\dfrac{x^2 + 3x + 2}{x^2 - x - 2}$ (b) $\dfrac{2x^2 - x - 1}{x^2 - 9} \cdot \dfrac{x + 3}{2x + 1}$

(c) $\dfrac{x^2}{x^2 - 4} - \dfrac{x + 1}{x + 2}$ (d) $\dfrac{\dfrac{y}{x} - \dfrac{x}{y}}{\dfrac{1}{y} - \dfrac{1}{x}}$

6. Rationalize the expression and simplify.

(a) $\dfrac{\sqrt{10}}{\sqrt{5}-2}$ (b) $\dfrac{\sqrt{4+h}-2}{h}$

7. Rewrite by completing the square.

(a) x^2+x+1 (b) $2x^2-12x+11$

8. Solve the equation. (Find only the real solutions.)

(a) $x+5=14-\frac{1}{2}x$ (b) $\dfrac{2x}{x+1}=\dfrac{2x-1}{x}$

(c) $x^2-x-12=0$ (d) $2x^2+4x+1=0$

(e) $x^4-3x^2+2=0$ (f) $3|x-4|=10$

(g) $2x(4-x)^{-1/2}-3\sqrt{4-x}=0$

9. Solve each inequality. Write your answer using interval notation.

(a) $-4<5-3x\leqslant 17$ (b) $x^2<2x+8$

(c) $x(x-1)(x+2)>0$ (d) $|x-4|<3$

(e) $\dfrac{2x-3}{x+1}\leqslant 1$

10. State whether each equation is true or false.

(a) $(p+q)^2=p^2+q^2$ (b) $\sqrt{ab}=\sqrt{a}\,\sqrt{b}$

(c) $\sqrt{a^2+b^2}=a+b$ (d) $\dfrac{1+TC}{C}=1+T$

(e) $\dfrac{1}{x-y}=\dfrac{1}{x}-\dfrac{1}{y}$ (f) $\dfrac{1/x}{a/x-b/x}=\dfrac{1}{a-b}$

ANSWERS TO DIAGNOSTIC TEST A: ALGEBRA

1. (a) 81 (b) -81 (c) $\frac{1}{81}$
(d) 25 (e) $\frac{9}{4}$ (f) $\frac{1}{8}$

2. (a) $6\sqrt{2}$ (b) $48a^5b^7$ (c) $\dfrac{x}{9y^7}$

3. (a) $11x-2$ (b) $4x^2+7x-15$
(c) $a-b$ (d) $4x^2+12x+9$
(e) $x^3+6x^2+12x+8$

4. (a) $(2x-5)(2x+5)$ (b) $(2x-3)(x+4)$
(c) $(x-3)(x-2)(x+2)$ (d) $x(x+3)(x^2-3x+9)$
(e) $3x^{-1/2}(x-1)(x-2)$ (f) $xy(x-2)(x+2)$

5. (a) $\dfrac{x+2}{x-2}$ (b) $\dfrac{x-1}{x-3}$
(c) $\dfrac{1}{x-2}$ (d) $-(x+y)$

6. (a) $5\sqrt{2}+2\sqrt{10}$ (b) $\dfrac{1}{\sqrt{4+h}+2}$

7. (a) $\left(x+\frac{1}{2}\right)^2+\frac{3}{4}$ (b) $2(x-3)^2-7$

8. (a) 6 (b) 1 (c) $-3, 4$
(d) $-1\pm\frac{1}{2}\sqrt{2}$ (e) $\pm 1, \pm\sqrt{2}$ (f) $\frac{2}{3}, \frac{22}{3}$
(g) $\frac{12}{5}$

9. (a) $[-4, 3)$ (b) $(-2, 4)$
(c) $(-2, 0)\cup(1, \infty)$ (d) $(1, 7)$
(e) $(-1, 4]$

10. (a) False (b) True (c) False
(d) False (e) False (f) True

If you have had difficulty with these problems, you may wish to consult the Review of Algebra on the website www.stewartcalculus.com.

C DIAGNOSTIC TEST: FUNCTIONS

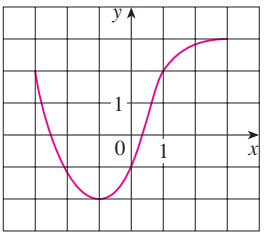


FIGURE FOR PROBLEM 1

1. The graph of a function f is given at the left.
(a) State the value of $f(-1)$.
(b) Estimate the value of $f(2)$.
(c) For what values of x is $f(x) = 2$?
(d) Estimate the values of x such that $f(x) = 0$.
(e) State the domain and range of f.

2. If $f(x) = x^3$, evaluate the difference quotient $\dfrac{f(2+h) - f(2)}{h}$ and simplify your answer.

3. Find the domain of the function.

(a) $f(x) = \dfrac{2x+1}{x^2+x-2}$ (b) $g(x) = \dfrac{\sqrt[3]{x}}{x^2+1}$ (c) $h(x) = \sqrt{4-x} + \sqrt{x^2-1}$

4. How are graphs of the functions obtained from the graph of f?

(a) $y = -f(x)$ (b) $y = 2f(x) - 1$ (c) $y = f(x-3) + 2$

5. Without using a calculator, make a rough sketch of the graph.

(a) $y = x^3$ (b) $y = (x+1)^3$ (c) $y = (x-2)^3 + 3$
(d) $y = 4 - x^2$ (e) $y = \sqrt{x}$ (f) $y = 2\sqrt{x}$
(g) $y = -2^x$ (h) $y = 1 + x^{-1}$

6. Let $f(x) = \begin{cases} 1 - x^2 & \text{if } x \leq 0 \\ 2x + 1 & \text{if } x > 0 \end{cases}$

(a) Evaluate $f(-2)$ and $f(1)$. (b) Sketch the graph of f.

7. If $f(x) = x^2 + 2x - 1$ and $g(x) = 2x - 3$, find each of the following functions.

(a) $f \circ g$ (b) $g \circ f$ (c) $g \circ g \circ g$

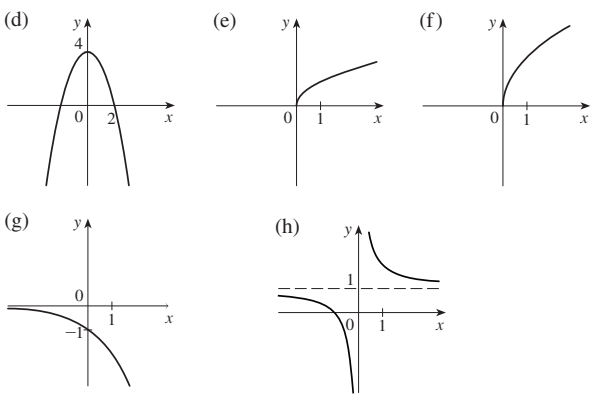
ANSWERS TO DIAGNOSTIC TEST C: FUNCTIONS

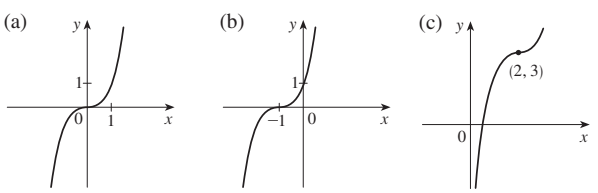
1. (a) -2 (b) 2.8
(c) $-3, 1$ (d) $-2.5, 0.3$
(e) $[-3, 3], [-2, 3]$

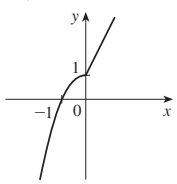
2. $12 + 6h + h^2$

3. (a) $(-\infty, -2) \cup (-2, 1) \cup (1, \infty)$
(b) $(-\infty, \infty)$
(c) $(-\infty, -1] \cup [1, 4]$

4. (a) Reflect about the x-axis
(b) Stretch vertically by a factor of 2, then shift 1 unit downward
(c) Shift 3 units to the right and 2 units upward



5.


6. (a) $-3, 3$
(b)


7. (a) $(f \circ g)(x) = 4x^2 - 8x + 2$
(b) $(g \circ f)(x) = 2x^2 + 4x - 5$
(c) $(g \circ g \circ g)(x) = 8x - 21$

If you have had difficulty with these problems, you should look at Sections 1.1–1.3 of this book.

D DIAGNOSTIC TEST: TRIGONOMETRY

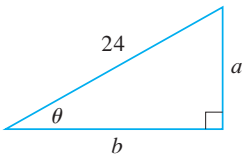


FIGURE FOR PROBLEM 5

1. Convert from degrees to radians.
(a) $300°$ (b) $-18°$

2. Convert from radians to degrees.
(a) $5\pi/6$ (b) 2

3. Find the length of an arc of a circle with radius 12 cm if the arc subtends a central angle of $30°$.

4. Find the exact values.
(a) $\tan(\pi/3)$ (b) $\sin(7\pi/6)$ (c) $\sec(5\pi/3)$

5. Express the lengths a and b in the figure in terms of θ.

6. If $\sin x = \frac{1}{3}$ and $\sec y = \frac{5}{4}$, where x and y lie between 0 and $\pi/2$, evaluate $\sin(x + y)$.

7. Prove the identities.
(a) $\tan\theta \sin\theta + \cos\theta = \sec\theta$
(b) $\dfrac{2\tan x}{1 + \tan^2 x} = \sin 2x$

8. Find all values of x such that $\sin 2x = \sin x$ and $0 \leqslant x \leqslant 2\pi$.

9. Sketch the graph of the function $y = 1 + \sin 2x$ without using a calculator.

ANSWERS TO DIAGNOSTIC TEST D: TRIGONOMETRY

1. (a) $5\pi/3$ (b) $-\pi/10$

2. (a) $150°$ (b) $360/\pi \approx 114.6°$

3. 2π cm

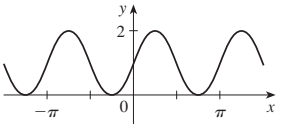
4. (a) $\sqrt{3}$ (b) $-\frac{1}{2}$ (c) 2

5. (a) $24\sin\theta$ (b) $24\cos\theta$

6. $\frac{1}{15}\left(4 + 6\sqrt{2}\right)$

8. $0, \pi/3, \pi, 5\pi/3, 2\pi$

9.



If you have had difficulty with these problems, you should look at Appendix D of this book.

EXERCISES

1–50 Calculate y'.

1. $y = (x^4 - 3x^2 + 5)^3$

2. $y = \cos(\tan x)$

3. $y = \sqrt{x} + \dfrac{1}{\sqrt[3]{x^4}}$

4. $y = \dfrac{3x - 2}{\sqrt{2x + 1}}$

5. $y = 2x\sqrt{x^2 + 1}$

6. $y = \dfrac{e^x}{1 + x^2}$

7. $y = e^{\sin 2\theta}$

8. $y = e^{-t}(t^2 - 2t + 2)$

9. $y = \dfrac{t}{1 - t^2}$

10. $y = e^{mx}\cos nx$

11. $y = \sqrt{x}\cos\sqrt{x}$

12. $y = (\arcsin 2x)^2$

13. $y = \dfrac{e^{1/x}}{x^2}$

14. $y = \dfrac{1}{\sin(x - \sin x)}$

15. $xy^4 + x^2y = x + 3y$

16. $y = \ln(\csc 5x)$

17. $y = \dfrac{\sec 2\theta}{1 + \tan 2\theta}$

18. $x^2\cos y + \sin 2y = xy$

19. $y = e^{cx}(c\sin x - \cos x)$

20. $y = \ln(x^2e^x)$

21. $y = 3^{x\ln x}$

22. $y = \sec(1 + x^2)$

23. $y = (1 - x^{-1})^{-1}$

24. $y = 1/\sqrt[3]{x + \sqrt{x}}$

25. $\sin(xy) = x^2 - y$

26. $y = \sqrt{\sin\sqrt{x}}$

27. $y = \log_5(1 + 2x)$

28. $y = (\cos x)^x$

29. $y = \ln\sin x - \frac{1}{2}\sin^2 x$

30. $y = \dfrac{(x^2 + 1)^4}{(2x + 1)^3(3x - 1)^5}$

31. $y = x\tan^{-1}(4x)$

32. $y = e^{\cos x} + \cos(e^x)$

33. $y = \ln|\sec 5x + \tan 5x|$

34. $y = 10^{\tan \pi\theta}$

35. $y = \cot(3x^2 + 5)$

36. $y = \sqrt{t\ln(t^4)}$

37. $y = \sin(\tan\sqrt{1 + x^3})$

38. $y = \arctan(\arcsin\sqrt{x})$

39. $y = \tan^2(\sin\theta)$

40. $xe^y = y - 1$

41. $y = \dfrac{\sqrt{x + 1}\,(2 - x)^5}{(x + 3)^7}$

42. $y = \dfrac{(x + \lambda)^4}{x^4 + \lambda^4}$

43. $y = x\sinh(x^2)$

44. $y = \dfrac{\sin mx}{x}$

45. $y = \ln(\cosh 3x)$

46. $y = \ln\left|\dfrac{x^2 - 4}{2x + 5}\right|$

47. $y = \cosh^{-1}(\sinh x)$

48. $y = x\tanh^{-1}\sqrt{x}$

49. $y = \cos(e^{\sqrt{\tan 3x}})$

50. $y = \sin^2(\cos\sqrt{\sin \pi x})$

51. If $f(t) = \sqrt{4t + 1}$, find $f''(2)$.

52. If $g(\theta) = \theta\sin\theta$, find $g''(\pi/6)$.

53. Find y'' if $x^6 + y^6 = 1$.

54. Find $f^{(n)}(x)$ if $f(x) = 1/(2 - x)$.

55. Use mathematical induction (page 77) to show that if $f(x) = xe^x$, then $f^{(n)}(x) = (x + n)e^x$.

56. Evaluate $\lim_{t\to 0}\dfrac{t^3}{\tan^3(2t)}$.

57–59 Find an equation of the tangent to the curve at the given point.

57. $y = 4\sin^2 x, \quad (\pi/6, 1)$

58. $y = \dfrac{x^2 - 1}{x^2 + 1}, \quad (0, -1)$

59. $y = \sqrt{1 + 4\sin x}, \quad (0, 1)$

60–61 Find equations of the tangent line and normal line to the curve at the given point.

60. $x^2 + 4xy + y^2 = 13, \quad (2, 1)$

61. $y = (2 + x)e^{-x}, \quad (0, 2)$

62. If $f(x) = xe^{\sin x}$, find $f'(x)$. Graph f and f' on the same screen and comment.

63. (a) If $f(x) = x\sqrt{5 - x}$, find $f'(x)$.
(b) Find equations of the tangent lines to the curve $y = x\sqrt{5 - x}$ at the points $(1, 2)$ and $(4, 4)$.
(c) Illustrate part (b) by graphing the curve and tangent lines on the same screen.
(d) Check to see that your answer to part (a) is reasonable by comparing the graphs of f and f'.

64. (a) If $f(x) = 4x - \tan x$, $-\pi/2 < x < \pi/2$, find f' and f''.
(b) Check to see that your answers to part (a) are reasonable by comparing the graphs of f, f', and f''.

65. At what points on the curve $y = \sin x + \cos x$, $0 \leqslant x \leqslant 2\pi$, is the tangent line horizontal?

66. Find the points on the ellipse $x^2 + 2y^2 = 1$ where the tangent line has slope 1.

67. If $f(x) = (x - a)(x - b)(x - c)$, show that

$$\frac{f'(x)}{f(x)} = \frac{1}{x - a} + \frac{1}{x - b} + \frac{1}{x - c}$$

68. (a) By differentiating the double-angle formula

$$\cos 2x = \cos^2 x - \sin^2 x$$

obtain the double-angle formula for the sine function.
(b) By differentiating the addition formula

$$\sin(x + a) = \sin x\cos a + \cos x\sin a$$

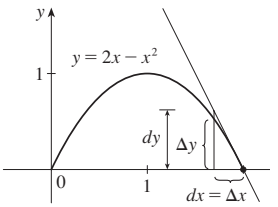
obtain the addition formula for the cosine function.

13. (a) $dy = \dfrac{-2}{(u-1)^2}\,du$ (b) $dy = -\dfrac{6r^2}{(1+r^3)^3}\,dr$

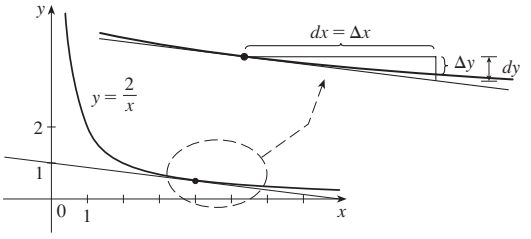
15. (a) $dy = \frac{1}{10}e^{x/10}\,dx$ (b) 0.01; 0.0101

17. (a) $dy = \sec^2 x\,dx$ (b) -0.2

19. $\Delta y = 0.64$, $dy = 0.8$



21. $\Delta y = -0.1$, $dy = -0.125$



23. 32.08 **25.** 4.02 **27.** $1 - \pi/90 \approx 0.965$

33. (a) 270 cm^3, 0.01, 1% (b) 36 cm^2, 0.00$\overline{6}$, 0.$\overline{6}$%

35. (a) $84/\pi \approx 27$ cm^2; $\frac{1}{84} \approx 0.012$

(b) $1764/\pi^2 \approx 179$ cm^3; $\frac{1}{56} \approx 0.018$

37. (a) $2\pi rh\,\Delta r$ (b) $\pi(\Delta r)^2 h$

43. (a) 4.8, 5.2 (b) Too large

EXERCISES 3.11 ■ PAGE 259

1. (a) 0 (b) 1 **3.** (a) $\frac{3}{4}$ (b) $\frac{1}{2}(e^2 - e^{-2}) \approx 3.62686$

5. (a) 1 (b) 0

21. $\operatorname{sech} x = \frac{3}{5}$, $\sinh x = \frac{4}{3}$, $\operatorname{csch} x = \frac{3}{4}$, $\tanh x = \frac{4}{5}$, $\coth x = \frac{5}{4}$

23. (a) 1 (b) -1 (c) ∞ (d) $-\infty$ (e) 0 (f) 1
(g) ∞ (h) $-\infty$ (i) 0

31. $f'(x) = x\cosh x$ **33.** $h'(x) = \tanh x$

35. $y' = 3e^{\cosh 3x}\sinh 3x$ **37.** $f'(t) = -2e^t \operatorname{sech}^2(e^t)\tanh(e^t)$

39. $y' = \dfrac{\operatorname{sech}^2 x}{1 + \tanh^2 x}$ **41.** $G'(x) = \dfrac{-2\sinh x}{(1+\cosh x)^2}$

43. $y' = \dfrac{1}{2\sqrt{x}(1-x)}$ **45.** $y' = \sinh^{-1}(x/3)$

47. $y' = \dfrac{-1}{x\sqrt{x^2+1}}$

51. (a) 0.3572 (b) 70.34°

53. (b) $y = 2\sinh 3x - 4\cosh 3x$

55. $\left(\ln(1+\sqrt{2}), \sqrt{2}\right)$

CHAPTER 3 REVIEW ■ PAGE 261

True-False Quiz

1. True **3.** True **5.** False **7.** False **9.** True
11. True

Exercises

1. $6x(x^4 - 3x^2 + 5)^2(2x^2 - 3)$ **3.** $\dfrac{1}{2\sqrt{x}} - \dfrac{4}{3\sqrt[3]{x^7}}$

5. $\dfrac{2(2x^2+1)}{\sqrt{x^2+1}}$ **7.** $2\cos 2\theta\, e^{\sin 2\theta}$

9. $\dfrac{t^2+1}{(1-t^2)^2}$ **11.** $\dfrac{\cos\sqrt{x} - \sqrt{x}\sin\sqrt{x}}{2\sqrt{x}}$

13. $-\dfrac{e^{1/x}(1+2x)}{x^4}$ **15.** $\dfrac{1 - y^4 - 2xy}{4xy^3 + x^2 - 3}$

17. $\dfrac{2\sec 2\theta\,(\tan 2\theta - 1)}{(1+\tan 2\theta)^2}$ **19.** $(1+c^2)e^{cx}\sin x$

21. $3^{x\ln x}(\ln 3)(1 + \ln x)$ **23.** $-(x-1)^{-2}$

25. $\dfrac{2x - y\cos(xy)}{x\cos(xy) + 1}$ **27.** $\dfrac{2}{(1+2x)\ln 5}$

29. $\cot x - \sin x\cos x$ **31.** $\dfrac{4x}{1+16x^2} + \tan^{-1}(4x)$

33. $5\sec 5x$ **35.** $-6x\csc^2(3x^2+5)$

37. $\cos\left(\tan\sqrt{1+x^3}\right)\left(\sec^2\sqrt{1+x^3}\right)\dfrac{3x^2}{2\sqrt{1+x^3}}$

39. $2\cos\theta\tan(\sin\theta)\sec^2(\sin\theta)$

41. $\dfrac{(x-2)^4(3x^2 - 55x - 52)}{2\sqrt{x+1}\,(x+3)^8}$ **43.** $2x^2\cosh(x^2) + \sinh(x^2)$

45. $3\tanh 3x$ **47.** $\dfrac{\cosh x}{\sqrt{\sinh^2 x - 1}}$

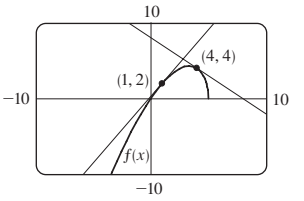
49. $\dfrac{-3\sin\left(e^{\sqrt{\tan 3x}}\right)e^{\sqrt{\tan 3x}}\sec^2(3x)}{2\sqrt{\tan 3x}}$ **51.** $-\frac{4}{27}$ **53.** $-5x^4/y^{11}$

57. $y = 2\sqrt{3}x + 1 - \pi\sqrt{3}/3$ **59.** $y = 2x + 1$

61. $y = -x + 2$; $y = x + 2$

63. (a) $\dfrac{10 - 3x}{2\sqrt{5-x}}$ (b) $y = \frac{7}{4}x + \frac{1}{4}$, $y = -x + 8$

(c)



65. $(\pi/4, \sqrt{2}), (5\pi/4, -\sqrt{2})$ **69.** (a) 2 (b) 44

71. $2xg(x) + x^2 g'(x)$ **73.** $2g(x)g'(x)$ **75.** $g'(e^x)e^x$

77. $g'(x)/g(x)$ **79.** $\dfrac{f'(x)[g(x)]^2 + g'(x)[f(x)]^2}{[f(x)+g(x)]^2}$

81. $f'(g(\sin 4x))g'(\sin 4x)(\cos 4x)(4)$ **83.** $(-3, 0)$

85. $y = -\frac{2}{3}x^2 + \frac{14}{3}x$

87. $v(t) = -Ae^{-ct}[c\cos(\omega t + \delta) + \omega\sin(\omega t + \delta)]$,
$a(t) = Ae^{-ct}[(c^2 - \omega^2)\cos(\omega t + \delta) + 2c\omega\sin(\omega t + \delta)]$

89. (a) $v(t) = 3t^2 - 12$; $a(t) = 6t$ (b) $t > 2$; $0 \leqslant t < 2$
(c) 23 (d) (e) $t > 2$; $0 < t < 2$