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On global regularity for involutive systems on compact manifolds

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In this talk we will deal with global hypoellipticity of involutive structures defined on a smooth product manifold $M \times \mathbb{T}^m$, where $\mathbb{T}^m \simeq \mathbb{R}^n/2\pi\mathbb{Z}^m$ is the m-dimensional torus.

Given a compact n-dimensional manifold M and closed 1-forms $\omega_1, \ldots, \omega_m$ over M, consider the product manifold $\Omega = M \times \mathbb{T}^m$ and the subbundle $\mathcal{V} \subset \mathbb{C}T\Omega$ annihilated by all the forms

$$\zeta_k = \mathsf{d}x_k - \omega_k, \quad k \in \{1, \ldots, m\},$$

where $(x_1, \ldots, x_m)$ are angular coordinates on $\mathbb{T}^m$. Such a bundle is involutive, and gives rise to a complex of vector bundles over Ω and first-order differential operators between them.

Let Λ^1 be the bundle of complex 1-forms over M, and $\Lambda^{0,1}$ be its pullback to Ω via the natural projection $\Omega \to M$; we denote their spaces of smooth sections by $\Lambda^1 C^\infty(M)$ and $\Lambda^{0,1}C^\infty(\Omega)$, respectively. We define $\mathbb{L} : C^\infty(\Omega) \to \Lambda^{0,1}C^\infty(\Omega)$ by the formula

$$\mathbb{L}f = \mathsf{d}_t f + \sum_{k=1}^{m} \omega_k \wedge \partial_{x_k} f,$$

where $t \in M$ and d_t is the exterior derivative on M; hence, $\mathbb{L}$ is a first-order differential operator.

We will investigate the regularity of solutions of the equation $\mathbb{L}u = f$.

We will give special attention in the case where M is also a torus $\mathbb{T}^n$; also, in the case where M is a compact Lie group.

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References

[1] G. Araújo, P. L. Dattori da Silva, and B. de Lessa Victor, Global Analytic Solvability of Involutive Systems on Compact Manifolds, *Journal of Geometric Analysis*, **33** (2023), 151.

[2] G. Araújo, P. L. Dattori da Silva, and B. de Lessa Victor, Global analytic hypoellipticity of involutive systems on compact manifolds, *Mathematische Annalen* (2022).

[3] P. L. Dattori da Silva and A. Meziani, A Gevrey differential complex on the torus, *Journal of Fourier Analysis and Applications*, **269**, 1 (2020), 8.

[4] I. V. Coelho da Silva, P. L. Dattori da Silva, and F. M. Simão, Regularity of solutions for a class of involutive systems of vector fields (preprint).