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**On non-autonomous exponential
attractors for a RD model**

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The following classical non-autonomous reaction-diffusion model is considered:

$$\begin{cases} \frac{\partial u}{\partial t} - \Delta u = f(u) + h(t) & \text{in } \Omega \times (\tau, \infty), \\ u|_{\partial\Omega} = 0 & \text{on } \partial\Omega \times (\tau, \infty), \\ u(\tau) = u_\tau & \text{in } \Omega, \end{cases}$$

where $\Omega \subset \mathbb{R}^N$ ($N \geq 1$) is an open bounded set (no regularity on the boundary is required), $\tau \in \mathbb{R}$, $u_\tau \in L^2(\Omega)$, and the time-dependent force satisfies $h \in L^2_{loc}(\mathbb{R}; H^{-1}(\Omega))$. Additional assumptions concerning the nonlinearity term f and h are given in two differentiated settings.

Firstly we establish the well-posedness of the process associated with the above problem under a nonlinearity f with certain growth condition depending on the dimension of the space and a translation bounded force h with values in $H^{-1}(\Omega)$, i.e. $h \in L^2_b(\mathbb{R}; H^{-1}(\Omega))$.

Our main goal is to ensure the existence of a pullback exponential attractor in $L^2(\Omega)$, and, consequently, the existence of a pullback attractor in $L^2(\Omega)$ with uniform finite fractal dimension of its sections. This is done using two different approaches, one by following the construction by Carvalho and Sonner [2, 3]; for the second one, the force h is relaxed to have exponential growth in the past, and there we follow the ideas developed in a previous work by Czaja [4].

This work has been done jointly with Álvaro Aguilar-Reyes (Universidad de Sevilla) (cf. [1]).

References

- [1] Á. Aguilar-Reyes and P. Marín-Rubio, Remarks on exponential attractors for a non-autonomous PDE with H^{-1} valued forces, *Commun. Pure Appl. Anal.*, **24** (2025), 1261–1279.

- [2] A. Carvalho and S. Sonner, Pullback exponential attractors for evolution processes in Banach spaces: theoretical results, *Commun. Pure Appl. Anal.*, **12** (2013), 3047-3071.
- [3] A. Carvalho and S. Sonner, Pullback exponential attractors for evolution processes in Banach spaces: properties and applications, *Commun. Pure Appl. Anal.*, **13** (2014), 1141-1165.
- [4] R. Czaja, Pullback exponential attractors with admissible exponential growth in the past, *Nonlinear Anal.*, **104** (2014), 90-108.