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On a climate energy balance model with nonlinear diffusion

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We are concerned with global climate models formulated in terms of partial differential equations.

Energy balance models (EBM) in global climate were introduced independently by M.I. Budyko [1] and W.D. Sellers [6]. Both models include the planetary coalbedo effect, which is the dominant feedback mechanism in the energy balance. We distinguish energy balance models by the choice of the Coalbedo function: Budyko-type models and Sellers-type models. It is well known that this type of model is very sensitive to the fluctuations in some parameters such as the Solar parameter Q. Most of the analyses of this type of energy balance model assume that the planetary coalbedo is monotone ([2],[5]). In this work we consider a non-monotone Coalbedo and consequently the model distinguishes different climatic zones (see [4]).

We also consider a coupled model surface-deep ocean effect, where an Energy Balance Model (EBM) is used to model the surface temperature and a parabolic equation in a global ocean with a dynamic and diffusive boundary condition represents the evolution of the deep ocean temperature. Although this model is based on that proposed by Watts & Morantine [7], it also incorporates other processes such as the nonlinear diffusion and the effect of coalbedo as a function of temperature (see [2] and [3]). We analyse the number of steady states of the model under study, using the solar constant as parameter. We find an interval of values of the solar constant Q where there exist at least three stationary solutions. Moreover, for such an interval of Q more than three stationary solutions are obtained numerically.

References

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