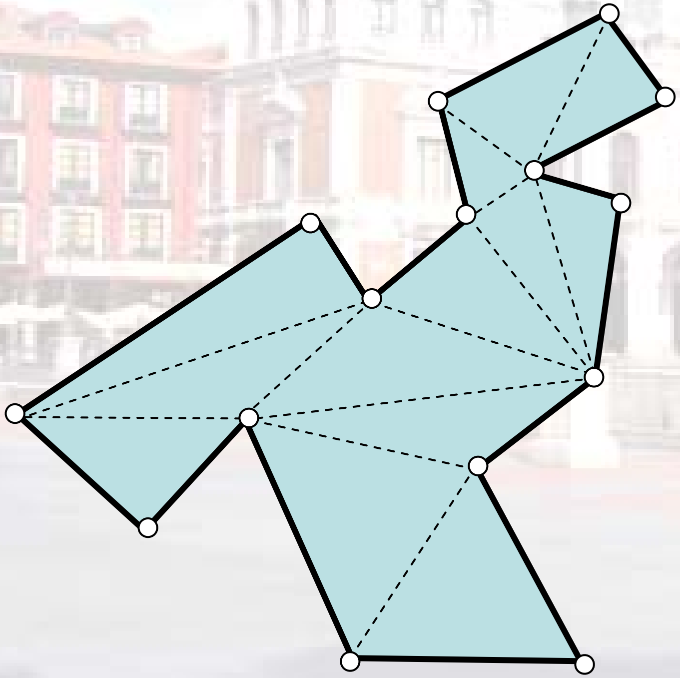
Monitoring triangulation graphs



Gregorio Hernández
Universidad Politécnica de Madrid, Spain

Overview

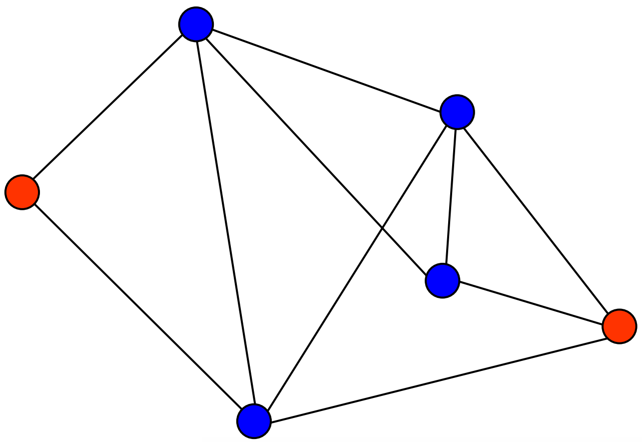
- Domination, covering, …, watching from faces.
  - Monitoring the elements of triangulations from its faces

- Extend the monitoring concepts to its distance versions for triangulation graphs

- Analyze monitoring concepts from a combinatorial point of view on **maximal outerplanar graphs**

- Analyze monitoring concepts from a combinatorial point of view on **triangulation graphs**

GRAPH THEORY



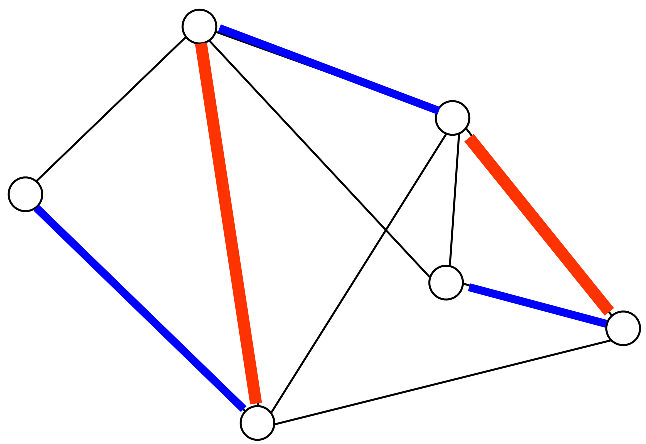
Controlling from vertices

- **DOMINATING SET**
- **VERTEX COVERING**

		Monitored Elements	
		Vertices	Edges
Monitored by	Vertices	Vertex Domination (Domination)	Vertex Covering (Covering)
	Edges	Edge Covering	Edge Domination

GRAPH THEORY

Controlling from edges

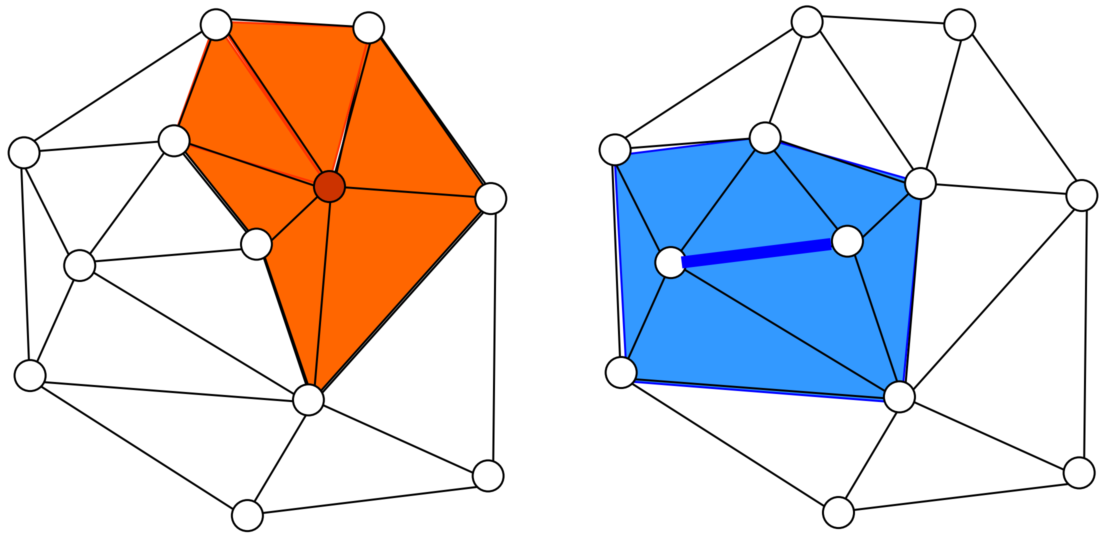


EDGE DOMINATING SET

EDGE COVERING

		Monitored Elements	
		Vertices	Edges
Monitored by	Vertices	Vertex Domination (Domination)	Vertex Covering (Covering)
	Edges	Edge Covering	Edge Domination

Triangulation graphs

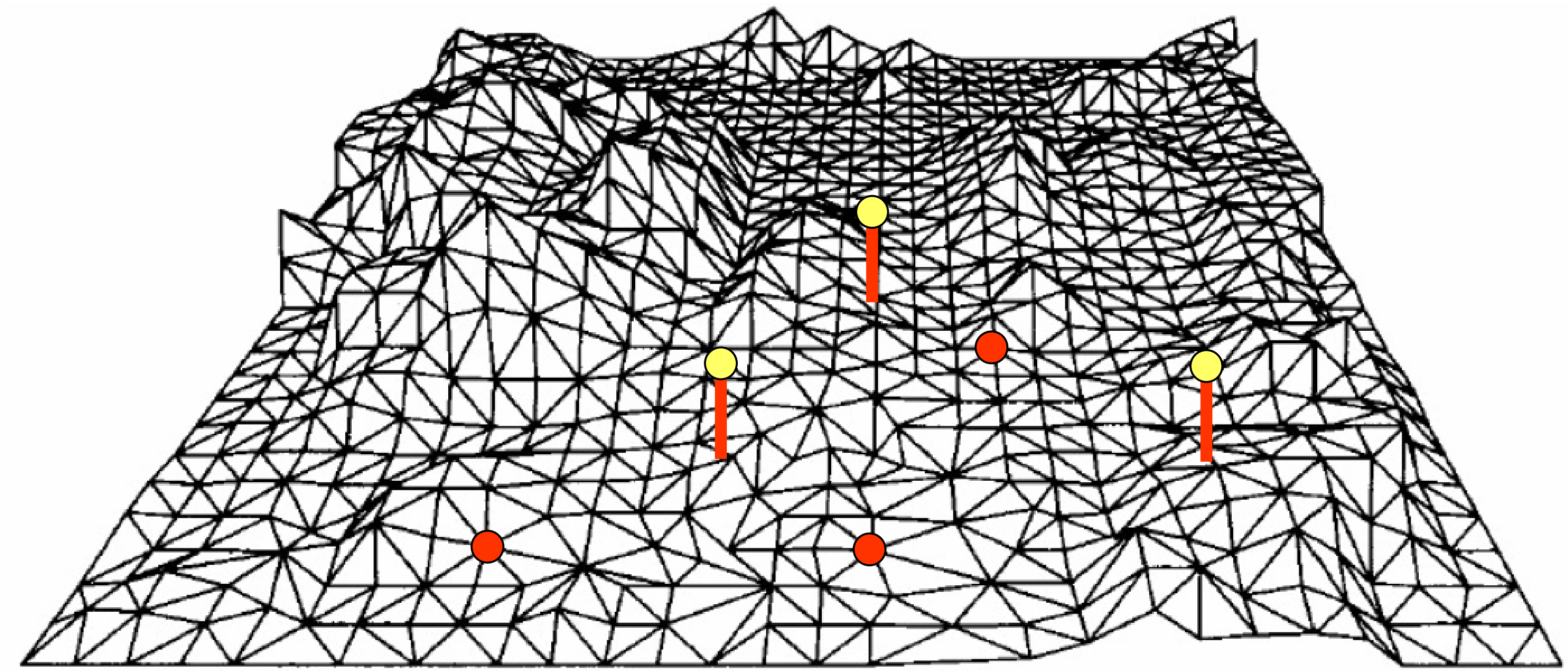


		Monitored Elements		
		Vertices	Faces	Edges
Monitored by	Vertices	Vertex Domination (Domination)	Vertex Guarding (Guarding)	Vertex Covering (Covering)
	Edges	Edge Covering	Edge Guarding	Edge Domination

COMPUTATIONAL GEOMETRY

TERRAIN GUARDING

How many guards?



Minimize is a NP-hard problem
Cole-Sharir, 89

VERTEX (POINT) GUARD
FIXED HEIGHT GUARD

TERRAIN GUARDING

How many guards?

Vertex guarding

$\left\lfloor \frac{n}{2} \right\rfloor$ vertices are always sufficient and sometimes necessary
Bose, Shermer, Toussaint, Zhu, 92

Edge guarding

$\left\lfloor \frac{n}{3} \right\rfloor$ edges are always sufficient (Everett, Rivera-Campo, 94)

$\left\lfloor \frac{4n-4}{13} \right\rfloor$ are sometimes necessary (BSTZ, 92, 97)

On triangulation graphs, we consider another monitoring concept (monitoring from faces)

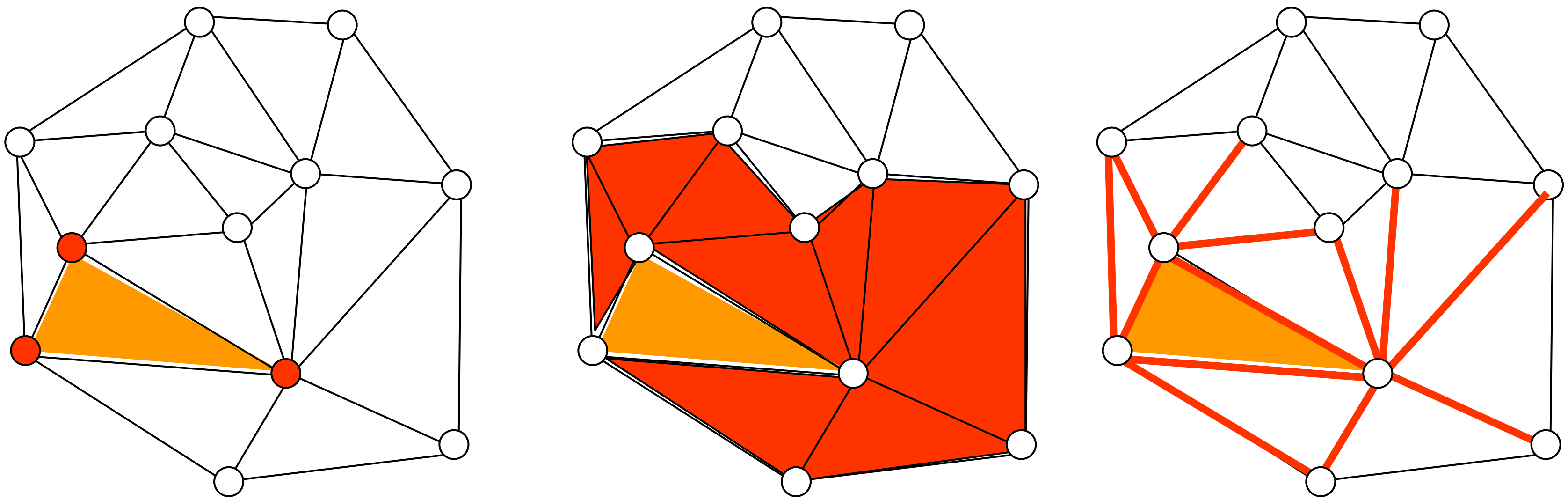
		Monitored Elements		
		Vertices	Faces	Edges
Monitored by	Vertices	Vertex Domination (Domination)	Vertex Guarding (Guarding)	Vertex Covering (Covering)
	Edges	Edge Covering	Edge Guarding	Edge Domination
	Faces	Face-vertex Covering	Face-face Guarding	Face-edge Covering

Watching from the faces (TRIANGULATIONS)

A triangle T_i face-vertex covers a vertex u if u is a vertex of T_i

A triangle T_i face guards T_k if they share some vertex

A triangle T_i face-edge covers an edge e if one of its endpoints is in T_i



MONITORING TRIANGULATIONS

Algorithmic aspects (from vertices)

Let be T a triangulation

$\gamma(T) = \min\{|D| \ / \ D \text{ is a dominant set of } T\}$

$g(T) = \min\{|G| \ / \ G \text{ is a set of guards of } T\}$

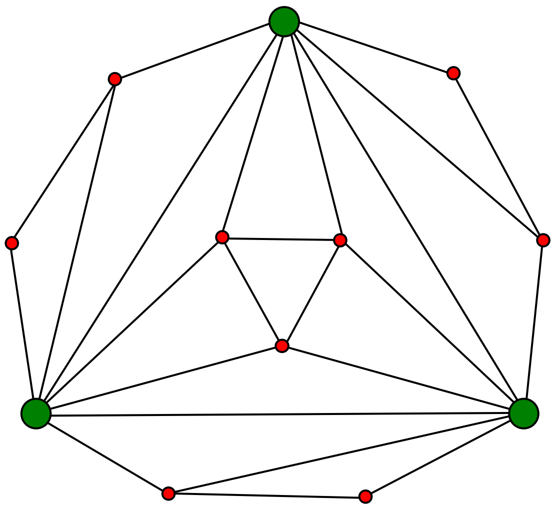
$\beta(T) = \min\{|K| \ / \ K \text{ is a vertex cover of } T\}$

Calculate these parameters are NP-complete problems

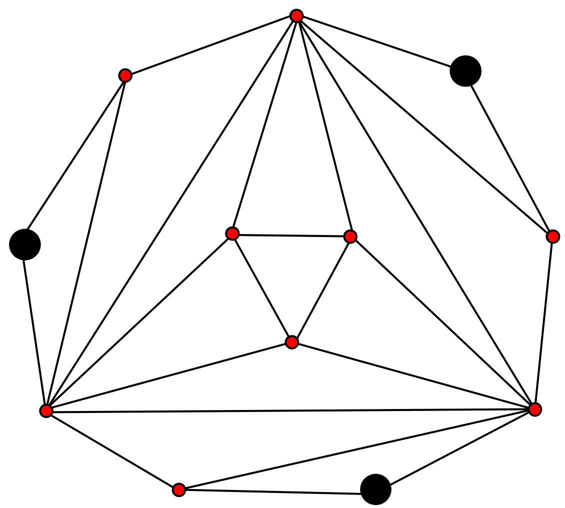
$$\gamma(T) \leq g(T) \leq \beta(T)$$

MONITORING TRIANGULATIONS

$\gamma(T) < g(T) < \beta(T)$



$\gamma(T) \leq 3$

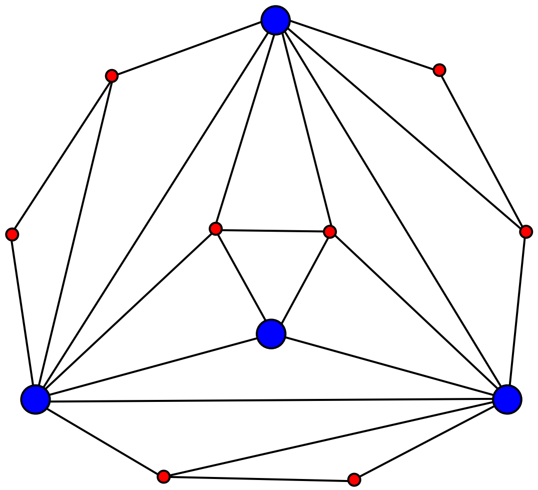


$\gamma(T) \geq 3$

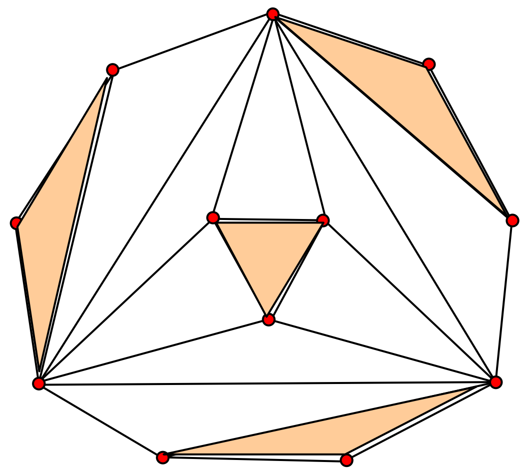
$\gamma(T) = 3$

MONITORING TRIANGULATIONS

$\gamma(T) < g(T) < \beta(T)$



$g(T) \leq 4$

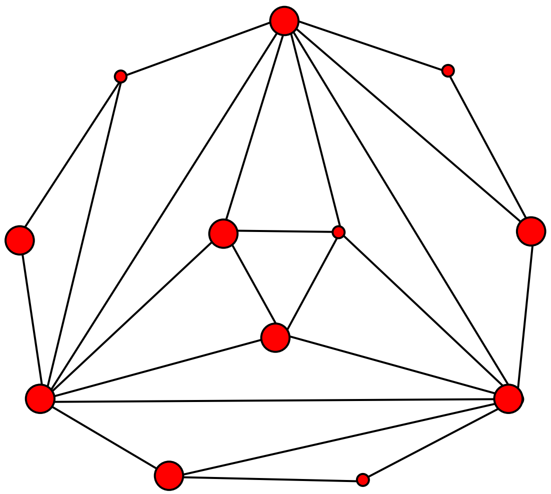


$g(T) \geq 4$

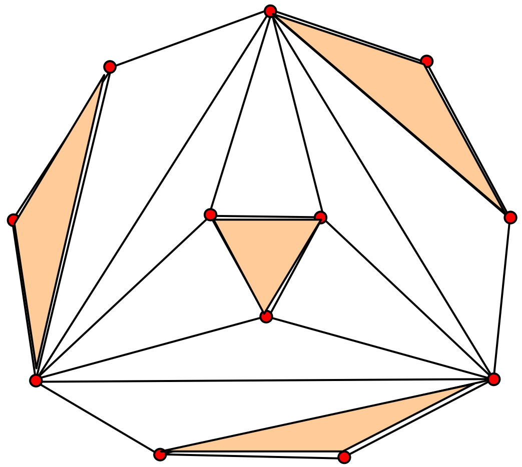
$g(T) = 4$

MONITORING TRIANGULATIONS

$\gamma(T) < g(T) < \beta(T)$



$\beta(T) \leq 8$



$\beta(T) \geq 8$

$\beta(T) = 8$

MONITORING TRIANGULATIONS

Combinatorial aspects

h(T) = min{|K| : K is a (-------) set of T}

(-------) dominant, guarding, vertex covering,
edge covering, edge guarding, edge dominating,
face-vertex covering, face guarding, face-edge-covering

h(n) = max {h(T) : T is a triangulation, T = (V,E) , |V| = n}

Combinatorial bounds for h(n)

MONITORING TRIANGULATION GRAPHS

		Watched elements		
		Vertices	Faces	Edges
Watching from	Vertices	Dominating $\gamma(n) = \left\lfloor \frac{n}{3} \right\rfloor$ (1)	Guarding $g(n) = \left\lfloor \frac{n}{2} \right\rfloor$ (2)	**Covering**
	Edges	**Edge-covering**	Edge-guarding $g^e(n) \leq \left\lfloor \frac{n}{3} \right\rfloor$ (3)	**Edge-dominating**
	Faces	**Face-vertex cover**	**Face-guarding**	**Face-edge cover**

(1) Matheson, Tarjan '96, (2) Bose et al. '97, (3) Everett, Rivera, '97

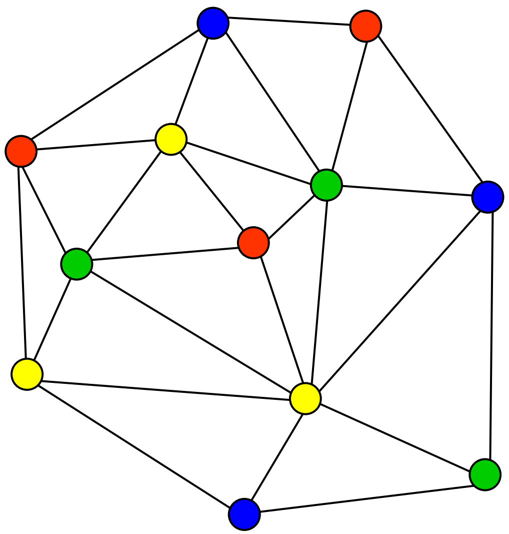
MONITORING TRIANGULATION GRAPHS

Watching from		Watched elements: Vertices	Watched elements: Faces	Watched elements: Edges
Watching from	Vertices	Dominating $\gamma(n) = \left\lfloor \frac{n}{3} \right\rfloor$ (1)	Guarding $g(n) = \left\lfloor \frac{n}{2} \right\rfloor$ (2)	**Covering** $\beta(n) = \left\lfloor \frac{3n}{4} \right\rfloor$
	Edges	**Edge-covering** $\beta'(n) = \left\lfloor \frac{2n-2}{3} \right\rfloor$	Edge-guarding $g^e(n) \leq \left\lfloor \frac{n}{3} \right\rfloor$ (3)	**Edge-dominating** $\left\lfloor \frac{2n-2}{5} \right\rfloor \leq \gamma'(n) \leq \left\lfloor \frac{n}{2} \right\rfloor$
	Faces	**Face-vertex cover** $f^v(n) = \left\lfloor \frac{2n-2}{3} \right\rfloor$	**Face-guarding** $\left\lfloor \frac{2n-2}{7} \right\rfloor \leq g^f(n) \leq \left\lfloor \frac{n}{3} \right\rfloor$	**Face-edge cover** ?

11th IWCG, Palencia 2015 (Flores, H., Orden, Seara, Urrutia)

VERTEX COVERING

Every n-vertex triangulation graph can be covered by $\left\lfloor \frac{3n}{4} \right\rfloor$ vertices and this bound is tight.



4-coloring vertices

VERTEX COVERING

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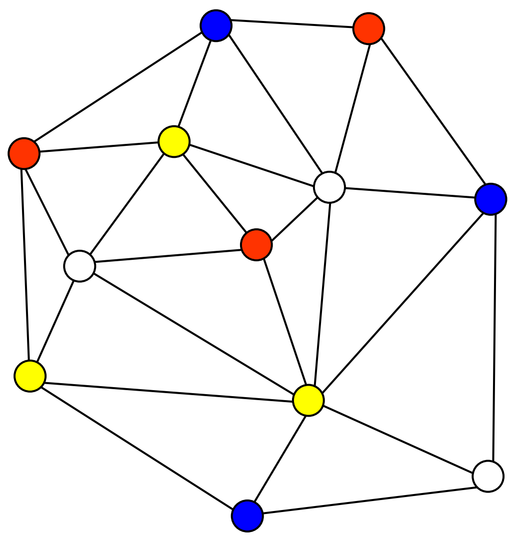


4-coloring vertices

Choose three colours less used

$\left\lfloor \frac{3n}{4} \right\rfloor$ vertices cover all edges

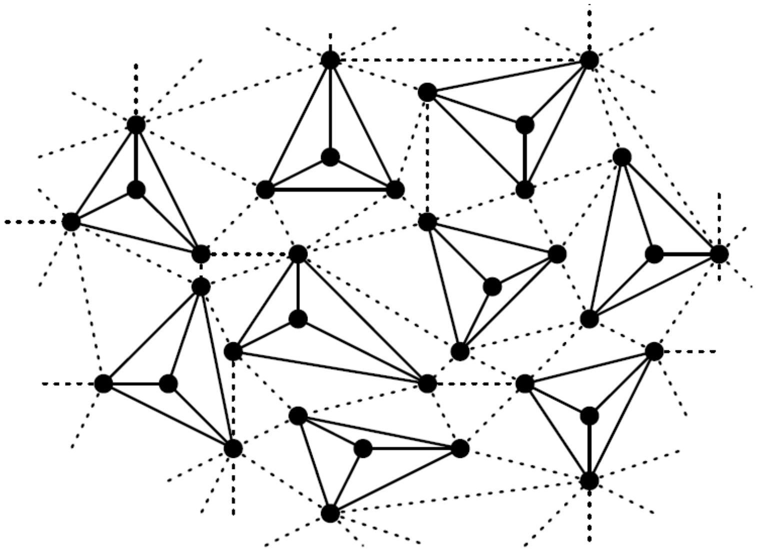
then $\beta'(T) \leq \left\lfloor \frac{3n}{4} \right\rfloor \quad \forall T$

$$\beta'(n) \leq \left\lfloor \frac{3n}{4} \right\rfloor$$

VERTEX COVERING

Every n-vertex triangulation graph can be covered by $\left\lfloor \frac{3n}{4} \right\rfloor$ vertices and this bound is tight.

Now, the lower bound



The edges of each K_4 need three different vertices to be covered, then

$$\left\lfloor \frac{3n}{4} \right\rfloor \leq \beta'(T)$$

Therefore,

$$\beta'(n) = \left\lfloor \frac{3n}{4} \right\rfloor$$

EDGE COVERING

Every n-vertex triangulation graph can be covered by $\left\lfloor \frac{2n-2}{3} \right\rfloor$ vertices and this bound is tight.

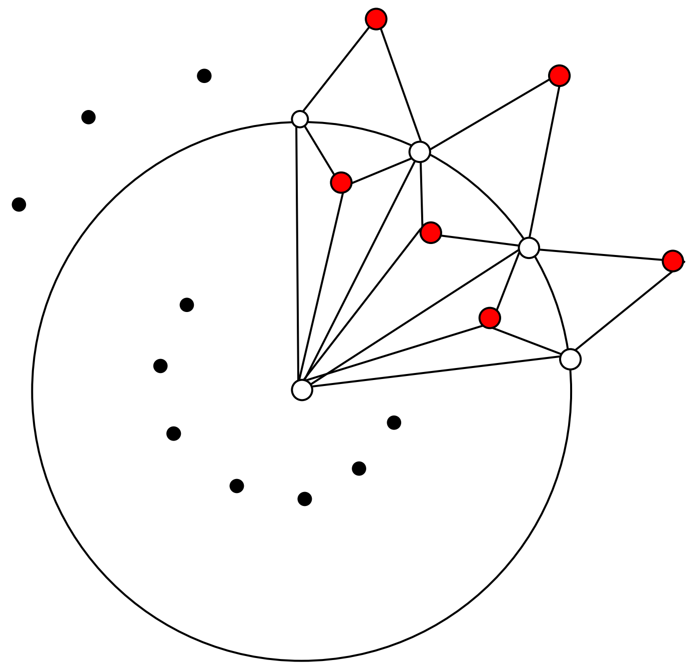
First, the lower bound

In the figure $n = k + k + k + 1$

The red vertices must be covered by different edges

$\beta'(T) \geq 2k$

Then $\left\lfloor \frac{2n-2}{3} \right\rfloor \leq \beta'(T)$



EDGE COVERING

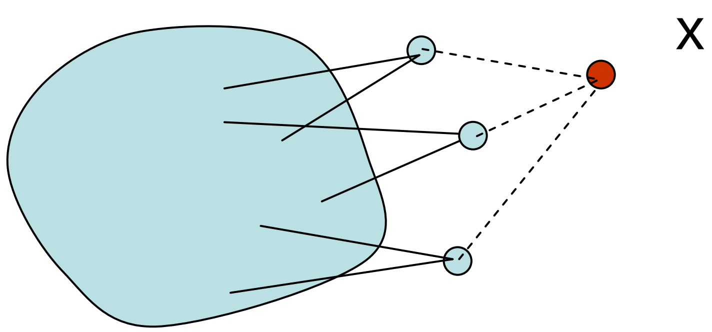
Every n-vertex triangulation graph can be covered by $\left\lfloor \frac{2n-2}{3} \right\rfloor$ vertices and this bound is tight.

Theorem (Nishizeki, '81)
G planar graph, 2-connected, $\delta \geq 3$, $n \geq 14$,
Then G contains a matching M so that $|M| \geq \left\lceil \frac{n+4}{3} \right\rceil$

Let be T triangulation. If there are vertices with degree 2
G*= T + x

G* has a matching M with

$$|M| \geq \left\lceil \frac{n+5}{3} \right\rceil$$

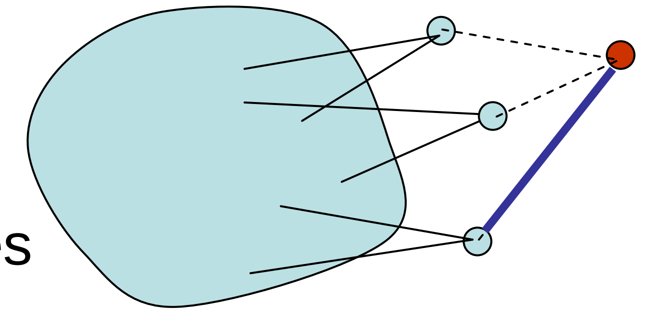


EDGE COVERING

Every n-vertex triangulation graph can be covered by $\left\lfloor \frac{2n-2}{3} \right\rfloor$ vertices and this bound is tight.

G*= T + x $\qquad |M| \geq \left\lceil \frac{n+5}{3} \right\rceil$

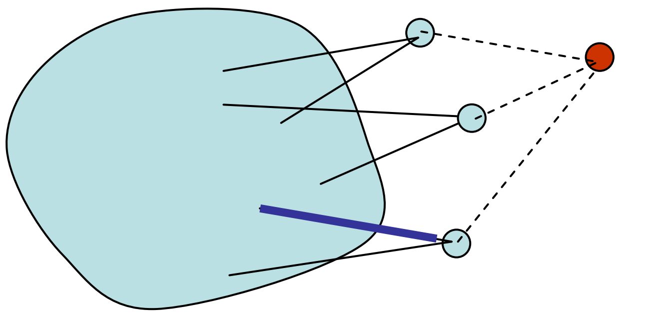
x



The edges of M cover $2\left\lceil \frac{n+5}{3} \right\rceil$ vertices

F = one edge for each free vertex in M

x



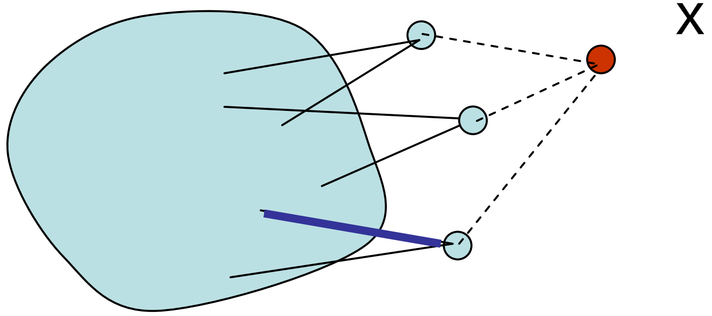
K=M∪F is an edge-covering of G*

K* is an edge-covering of T, |K*|=|K|

EDGE COVERING

Every n-vertex triangulation graph can be covered by $\left\lfloor \frac{2n-2}{3} \right\rfloor$ vertices and this bound is tight.

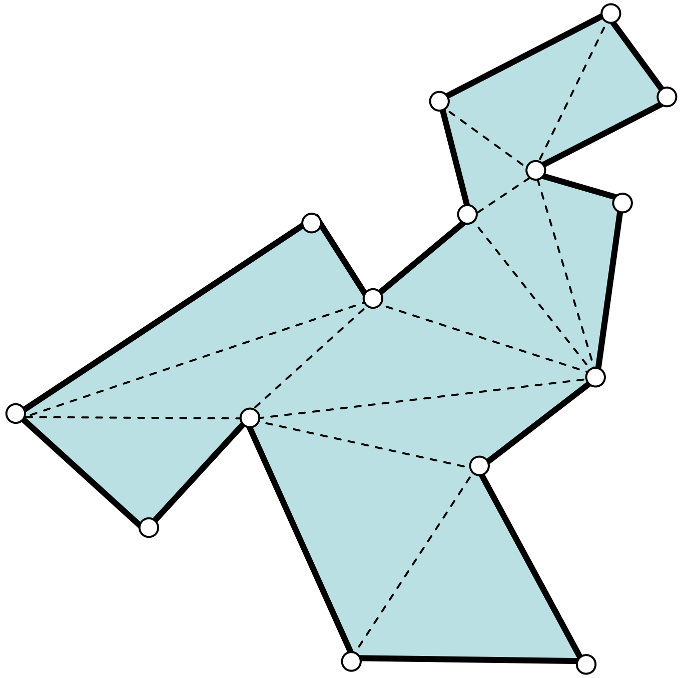
K* is an edge-covering of T, |K*|=|K|



$$|K^*| = \left\lceil \frac{n+5}{3} \right\rceil + (n+1) - 2\left\lceil \frac{n+5}{3} \right\rceil = n+1-\left\lceil \frac{n+5}{3} \right\rceil = \left\lfloor \frac{2n-2}{3} \right\rfloor$$

Therefore $\beta'(n) \leq \left\lfloor \frac{2n-2}{3} \right\rfloor$

MONITORING MAXIMAL OUTERPLANAR GRAPHS



Triangulation graph
without interior points

Triangulation graph of a polygon

h(n) = max {h(T) / T is a MOP, T = (V,E) , |V| = n}

MONITORING MAXIMAL OUTERPLANAR GRAPHS

Watching from	Watched elements: Vertices	Watched elements: Faces	Watched elements: Edges
Vertices	Dominating $\gamma(n) = \left\lfloor \frac{n+n_2}{4} \right\rfloor$ (1)	Guarding $g(n) = \left\lfloor \frac{n}{3} \right\rfloor$ (2)	**Covering**
Edges	**Edge-covering**	Edge-guarding $g^e(n) = \left\lfloor \frac{n}{4} \right\rfloor$ (3)	Edge-dominating $\gamma'(n) = \left\lfloor \frac{n+1}{3} \right\rfloor$ (4)
Faces	**Face-vertex cover**	**Face-guarding**	**Face-edge cover**

(1) Campos '13, (2) Art Gallery Theorem '76, (3) O´Rourke '83, (4) Karavelas '11

MONITORING MAXIMAL OUTERPLANAR GRAPHS

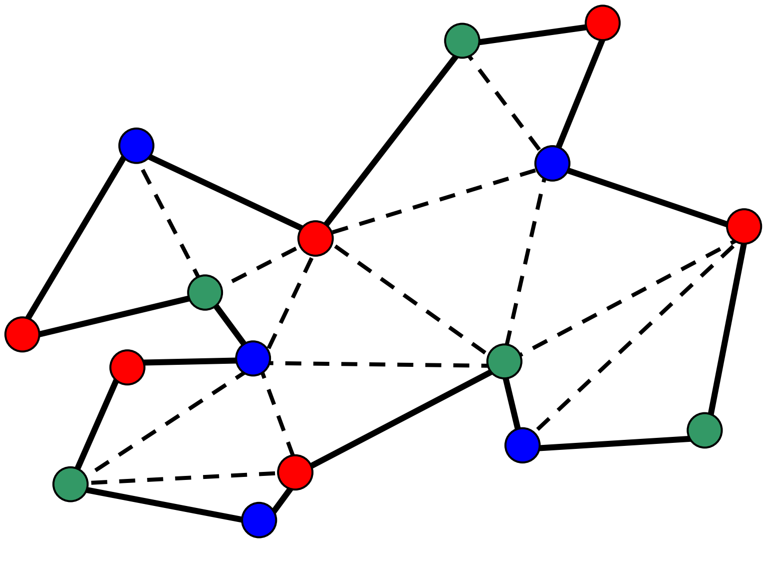
Watching from \ Watched elements	Vertices	Faces	Edges
Vertices	**Dominating** $\gamma(n)=\left\lfloor \frac{n+n_2}{4} \right\rfloor$ (1)	**Guarding** $g(n)=\left\lfloor \frac{n}{3} \right\rfloor$ (2)	**Covering** $\beta(n)=\left\lfloor \frac{2n}{3} \right\rfloor$
Edges	**Edge-covering** $\beta'(n)=\left\lfloor \frac{n+1}{2} \right\rfloor$	**Edge-guarding** $g^e(n)=\left\lfloor \frac{n}{4} \right\rfloor$ (3)	**Edge-dominating** $\gamma'(n)=\left\lfloor \frac{n+1}{3} \right\rfloor$ (4)
Faces	**Face-vertex cover** $f^v(n)=\left\lfloor \frac{n}{2} \right\rfloor$	**Face-guarding** $g^f(n)=\left\lfloor \frac{n}{4} \right\rfloor$	**Face-edge cover** $f^e(n)=\left\lfloor \frac{n}{3} \right\rfloor$

H., Martins '14,

VERTEX COVERING (en MOP's)

Every n-vertex maximal outerplanar graph can be covered by $\left\lfloor \frac{2n}{3} \right\rfloor$ vertices and this bound is tight.

3-coloring vertices



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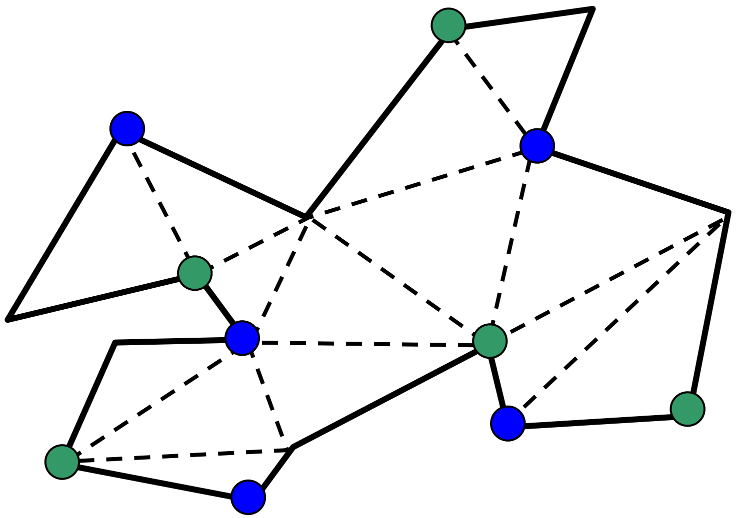


3-coloring vertices

Choose two colours less used

$\left\lfloor \frac{2n}{3} \right\rfloor$ vertices cover all edges

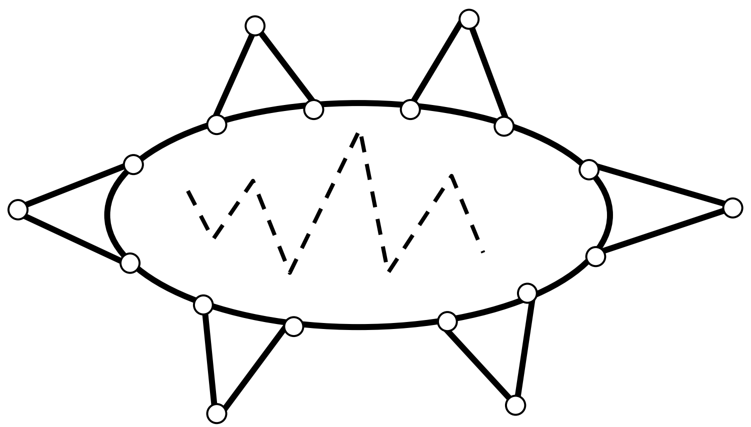
then $\beta(T) \leq \left\lfloor \frac{2n}{3} \right\rfloor \quad \forall T$

$$\beta(n) \leq \left\lfloor \frac{2n}{3} \right\rfloor$$

VERTEX COVERING (en MOP's)

Every n-vertex maximal outerplanar graph can be covered by $\left\lfloor \frac{2n}{3} \right\rfloor$ vertices and this bound is tight.

Now, the lower bound



The edges of each triangle need two different vertices to be covered, then

$$\left\lfloor \frac{2n}{3} \right\rfloor \leq \beta(T)$$

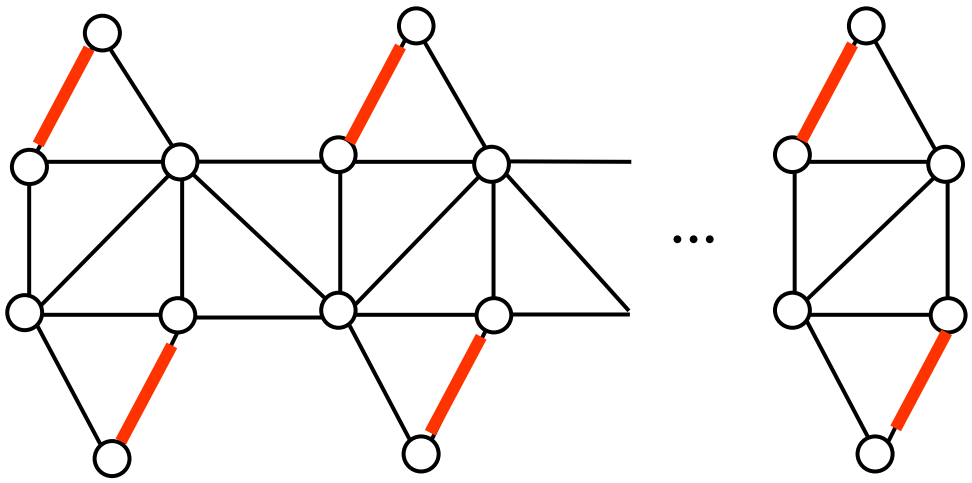
Therefore,

$$\beta(n) = \left\lfloor \frac{2n}{3} \right\rfloor$$

FACE-EDGE COVERING (en MOP's)

Every n-vertex maximal outerplanar graph, n≥4 can be face-edge covered by $\left\lfloor \frac{n}{3} \right\rfloor$ triangles (faces) and this bound is tight.

Lower bound



Red edges need different triangles to be face-covered, then

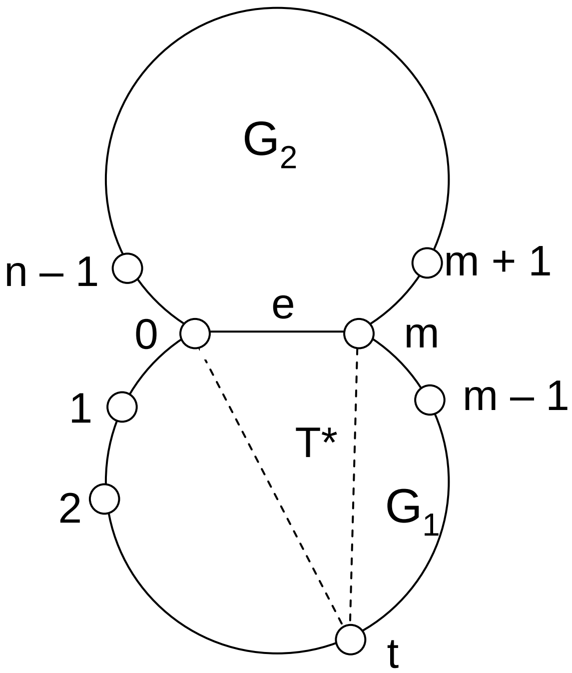
$$\left\lfloor \frac{n}{3} \right\rfloor \leq f^{e}(T)$$

FACE-EDGE COVERING (en MOP's)

Upper bound

Every n-vertex MOP T, can be face-edge covered by $\lfloor n/3 \rfloor$ faces

Lemma 1. Let T be a MOP with $n \geq 2s$ vertices. There is an interior edge e in T that separates off a minimum number m of exterior edges, where $m = s, s + 1, \ldots , 2s - 2$.



e diagonal of T that separates off a minimum number m of exterior edges which is at least s

$T^* = \triangle 0mt$

m is minimal ⇒ $\begin{cases} t \leq s - 1 \\ m - t \leq s - 1 \end{cases}$

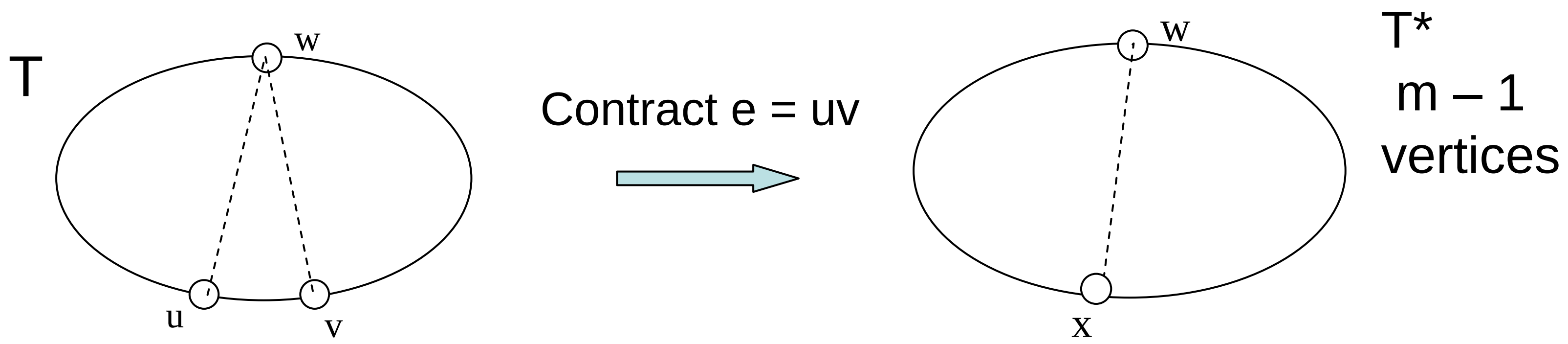
Then $m \leq 2s - 2$

FACE-EDGE COVERING (en MOP’s)

Upper bound

Every n-vertex MOP T, can be face-edge covered by $\lfloor n/3 \rfloor$ faces

Lemma 2. Suppose that f(m) triangles (faces) are always sufficient to cover the edges of any MOP T with m vertices. Let be e an exterior edge of T. Then f(m−1) triangles and an additional “collapsed triangle” at the edge e are sufficient to cover the edges of T.



T* is covered with f(m – 1) faces

FACE-EDGE COVERING (en MOP's)

Upper bound

Every n-vertex MOP T, can be face-edge covered by $\lfloor n/3 \rfloor$ faces

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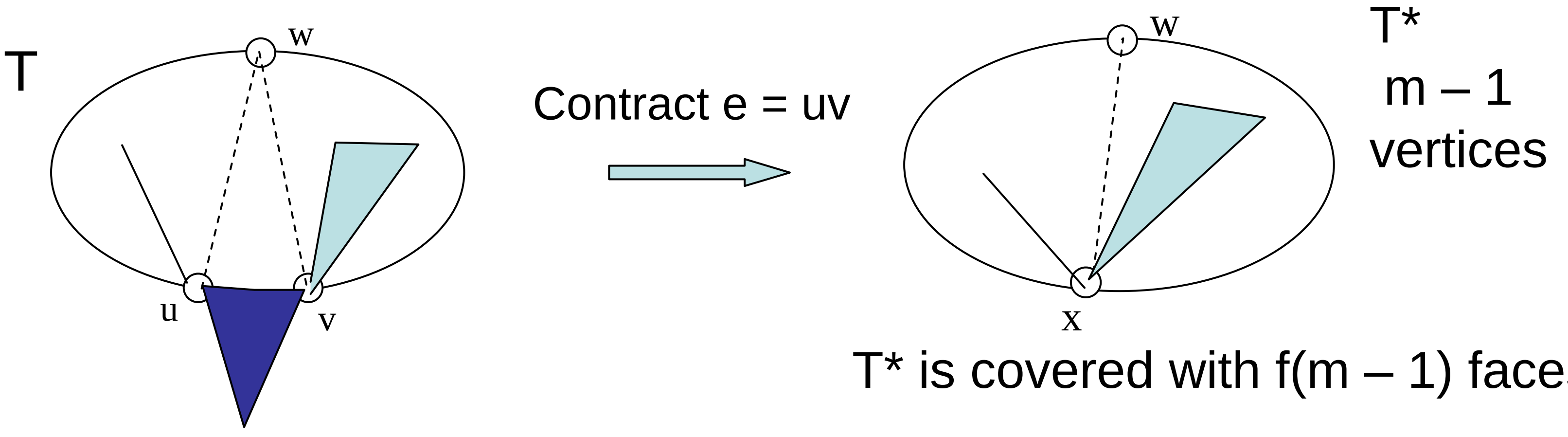


T* is covered with f(m – 1) faces

FACE-EDGE COVERING (en MOP's)

Upper bound

Every n-vertex MOP T, can be face-edge covered by $\lfloor n/3 \rfloor$ faces

Proof

Induction on n

Basic case: for $3 \leq n \leq 8$, easy

Inductive step: Let $n \geq 9$ and assume that the theorem holds for $n' < n$

Lemma 1 (s=4) guarantees the existence of a diagonal that divides T in G_1 and G_2, such that G_1 has m = 4, 5 or 6 exterior edges

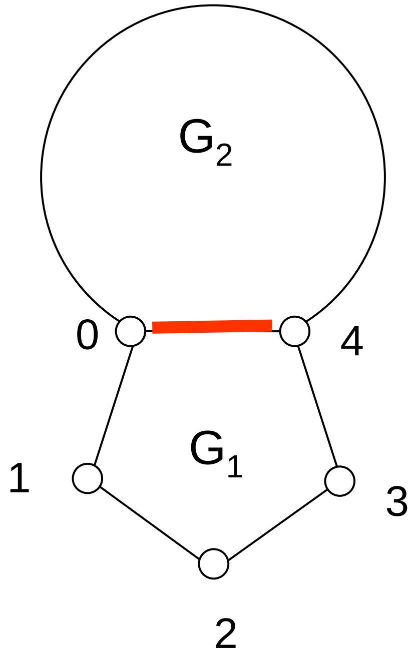
FACE-EDGE COVERING (en MOP's)

Upper bound

Every n-vertex MOP T, can be face-edge covered by $\lfloor n/3 \rfloor$ faces

Proof

Case m = 4



G_2 has n − 3 exterior edges

⇓ I.H.

can be face-covered with
□(n−3)/3□ = □n/3□−1
triangles

G_1 has 5 exterior edges

⇓

can be face-covered with one
triangle

G can be face-covered by □n/3□ faces

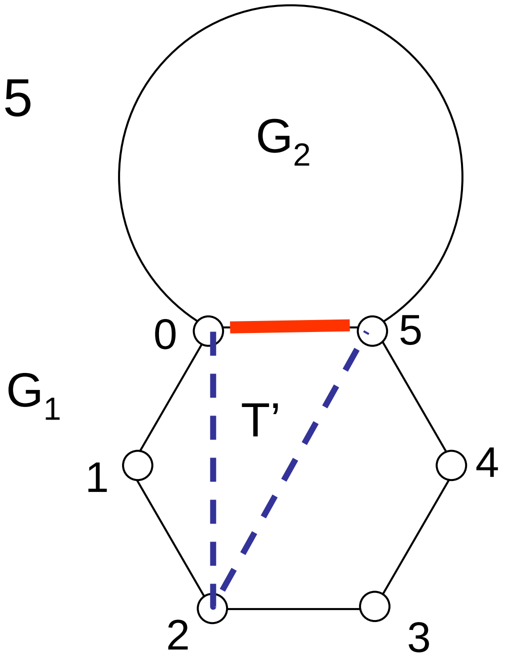
FACE-EDGE COVERING (en MOP's)

Upper bound

Every n-vertex MOP T, can be face-edge covered by $\lfloor n/3 \rfloor$ faces

Proof

Case m = 5



The presence of any of the internal edges (0,4) or (1,5) would violate the minimality of m

Thus, the triangle T' in G_1 that is bounded by e is (0,2,5) or (0,3,5)

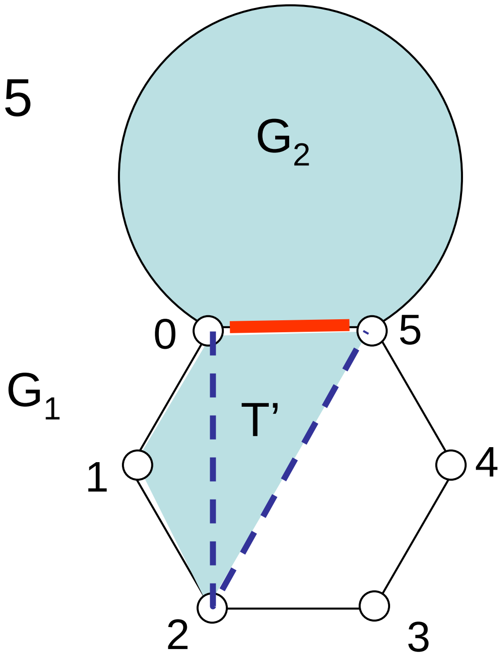
FACE-EDGE COVERING (en MOP's)

Upper bound

Every n-vertex MOP T, can be face-edge covered by $\lfloor n/3 \rfloor$ faces

Proof

Case m = 5



Consider T* = G_2 + 0125
T* is maximal outerplanar graph and has n – 2 vertices

By lemma 2 T* can be face-edge covered with $f(n-3) = \lfloor n/3 \rfloor - 1$ faces, and an additional "collapsed triangle" at the edge 25.

The "collapsed triangle" at 25, also face-covers the quadrilateral 2345, regardless how it is triangulated

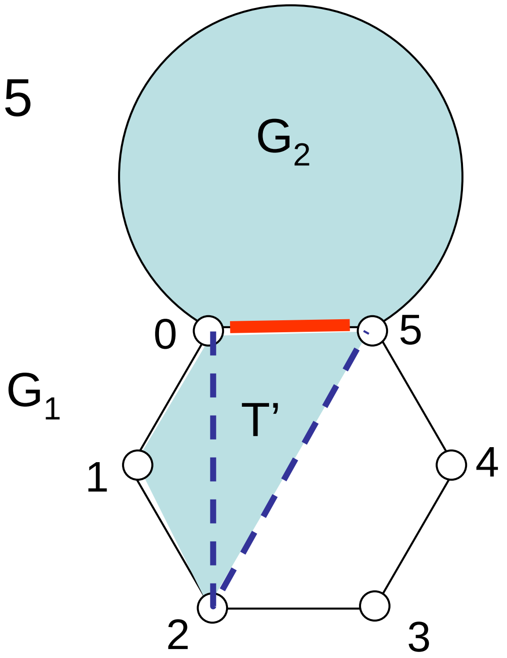
FACE-EDGE COVERING (en MOP's)

Upper bound

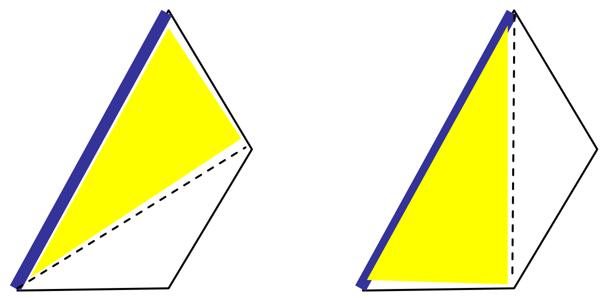
Every n-vertex MOP T, can be face-edge covered by $\lfloor n/3 \rfloor$ faces

Proof

Case m = 5



The "collapsed triangle" at 25, also face-covers the quadrilateral 2345, regardless how it is triangulated



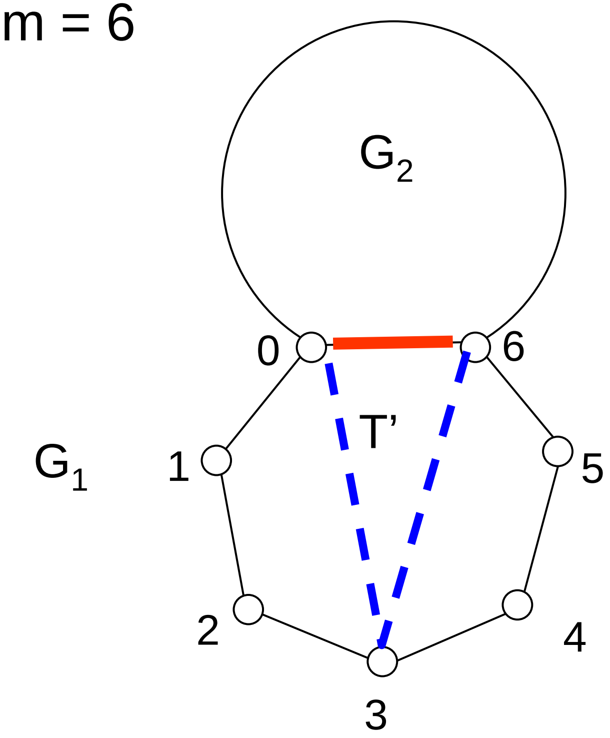
Therefore, T is face-edge covered by $\left\lfloor \frac{n}{3} \right\rfloor$ faces

FACE-EDGE COVERING (en MOP's)

Upper bound

Every n-vertex MOP T, can be face-edge covered by $\lfloor n/3 \rfloor$ faces

Case m = 6



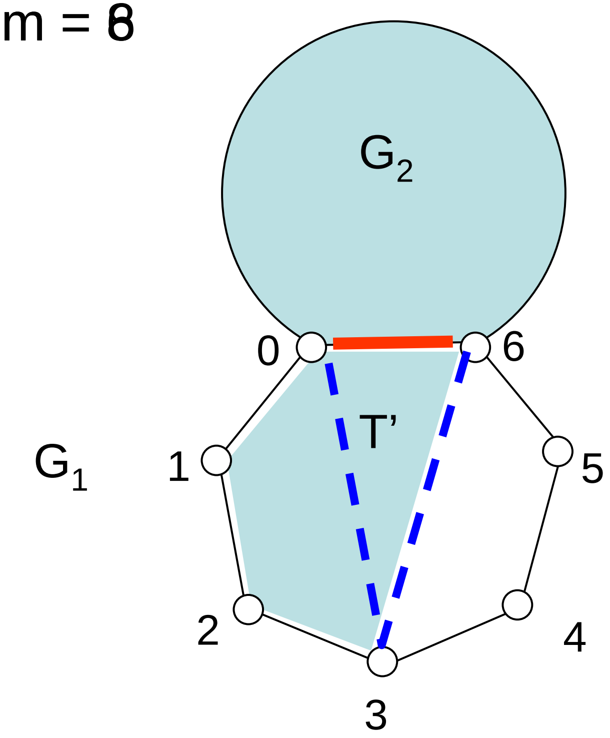
The presence of any of the internal edges (0,5), (0,4), (6,1) and (6,2) would violate the minimality of m

Thus, the triangle T' in G_1 that is bounded by e is (0,3,6)

Upper bound

Every n-vertex MOP T, can be face-edge covered by $\lfloor n/3 \rfloor$ faces

Case m = 6



Consider $T^* = G_2 + 01236$
T* is maximal outerplanar graph and has n – 2 vertices

By lemma 2 T* can be face-edge covered with $f(n-3) = \lfloor n/3 \rfloor - 1$ triangles, and an additional "collapsed triangle" at the edge 36 which covers 3456

Therefore, $\left\lfloor \frac{n}{3} \right\rfloor$ faces cover T

REMOTE MONITORIZATION

On triangulation graphs, we extended some monitoring concepts to its distance versions.

		Monitored Elements		
		Vertices	Faces	Edges
Monitored by	Vertices	kd-Vertex Domination (kd-Domination)	kd-Vertex Guarding (kd-Guarding)	kd-Vertex Covering (kd-Covering)
	Edges	kd-Edge Covering	kd-Edge Guarding	kd-Edge Domination
	Faces	kd-Face-vertex Covering	kd-Face-face Guarding	kd-Face-edge Covering

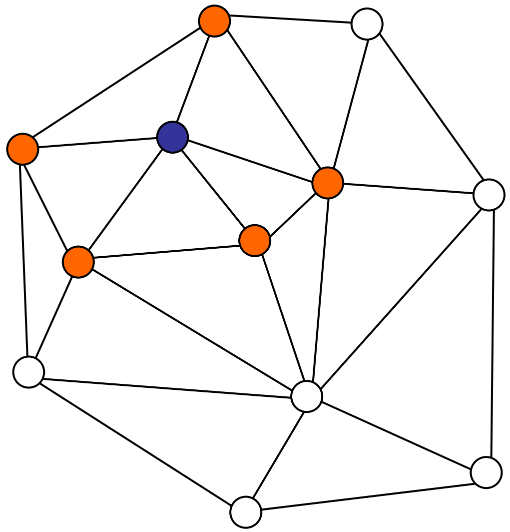
2013, Canales, H., Martins, Matos: "Distance domination, guarding and vertex cover for maximal outerplanar graphs"

REMOTE MONITORING BY VERTICES

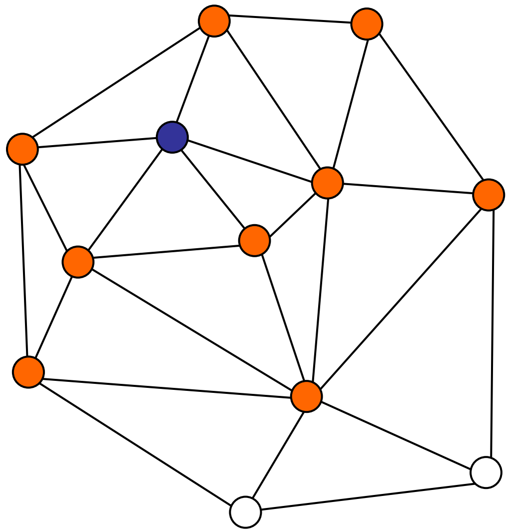
T=(V,E) triangulation graph

Distance k-domination

A vertex v kd-dominates a vertex u if $\text{dist}_T (v, u) \leq k$



k = 1 domination

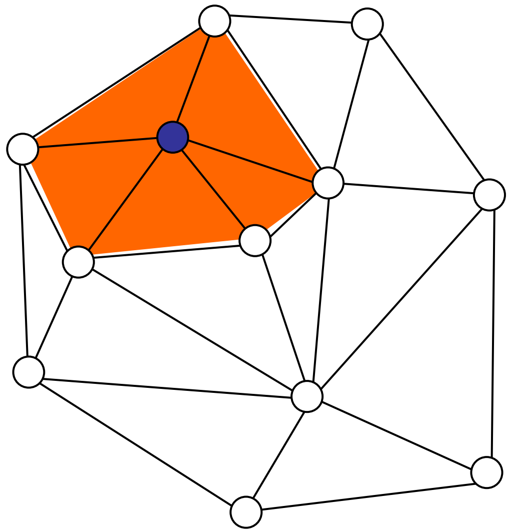


k = 2 2d-domination

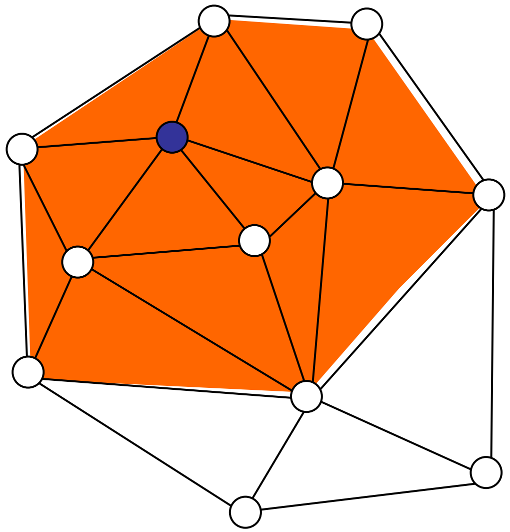
T=(V,E) triangulation graph

Guarding k-distance

A vertex v kd-guards a triangle T_i if $\text{dist}_T(v, T_i) \leq k - 1$



k = 1 guarding



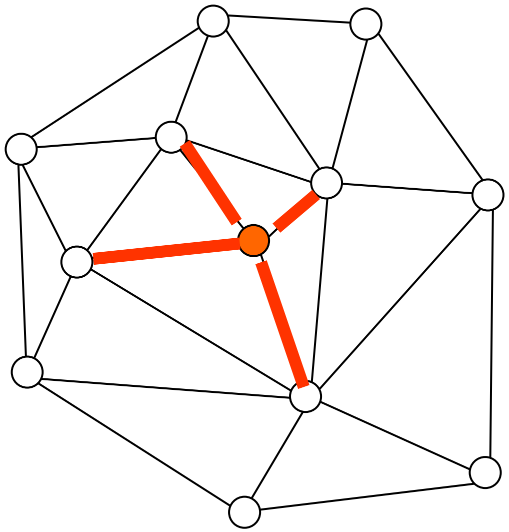
k = 2 2d-guarding

REMOTE MONITORING BY VERTICES

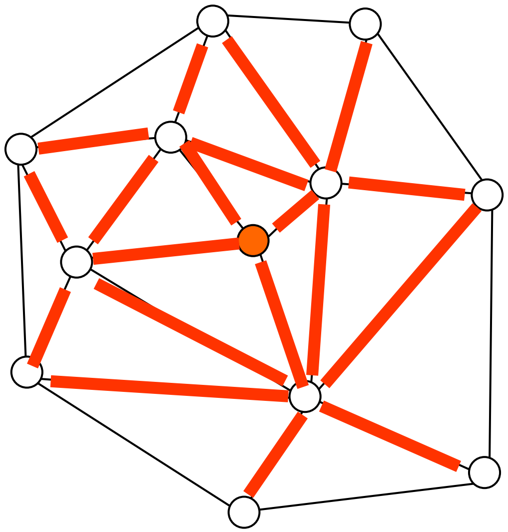
T=(V,E) triangulation graph

Vertex-covering k-distance

A vertex v kd-covers an edge e if $dist_T (v, e) \leq k - 1$



k = 1 vertex-covering



k = 2

REMOTE MONITORING

T=(V,E) triangulation graph

h_{kd} (T) = min{ |M| / M is a (-------) set of T}

(-------) distance k-dominating, k-guarding, k-vertex covering

$\gamma_{kd}(T)$, $g_{kd}(T)$, $\beta_{kd}(T)$

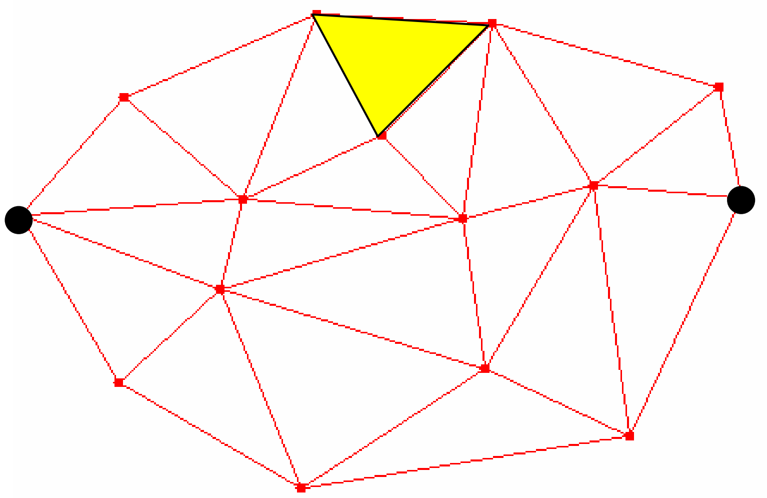
Algorithmic aspects

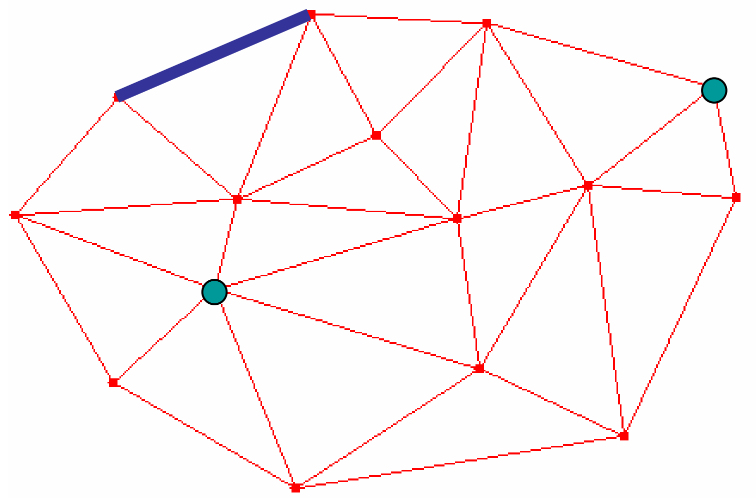
NP-complete problems

REMOTE MONITORING (distance 2)

T=(V,E) triangulation graph

$\gamma_{2d}(T) \leq g_{2d}(T) \leq \beta_{2d}(T)$





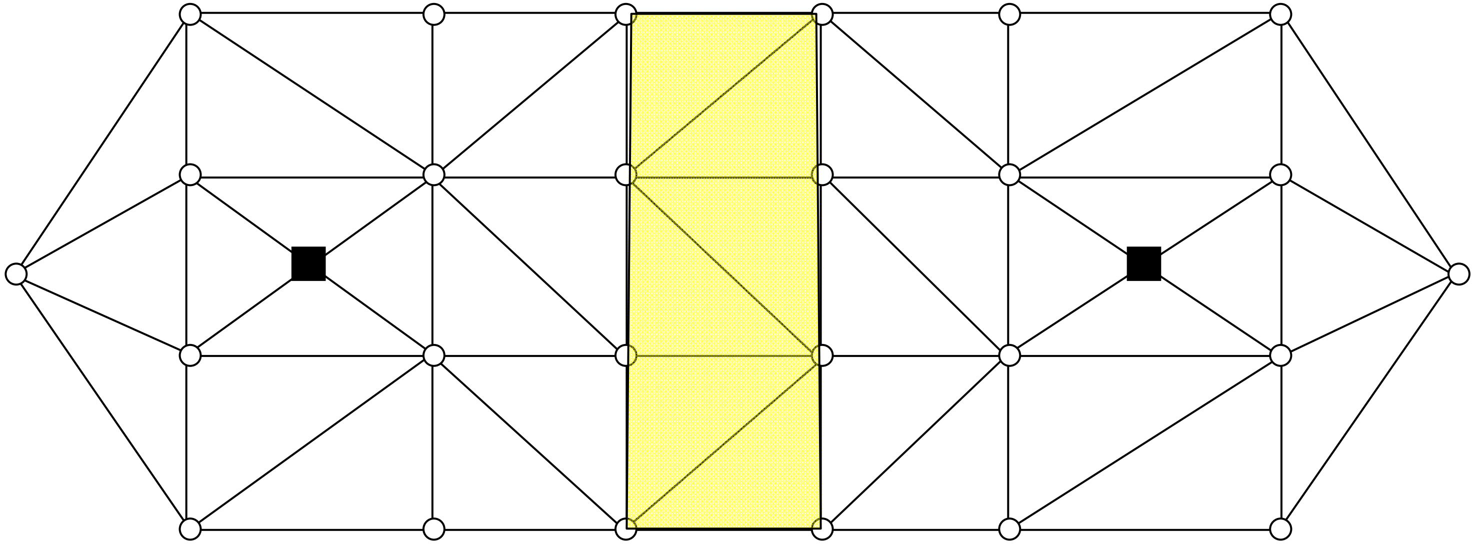
D = {●}
2d-dominating set
not 2d-guarding

G = {●}
2d-guarding set
not 2d-vertex cover

REMOTE MONITORING (distance 2)

T=(V,E) triangulation graph

$\gamma_{2d}(T) < g_{2d}(T) < \beta_{2d}(T)$



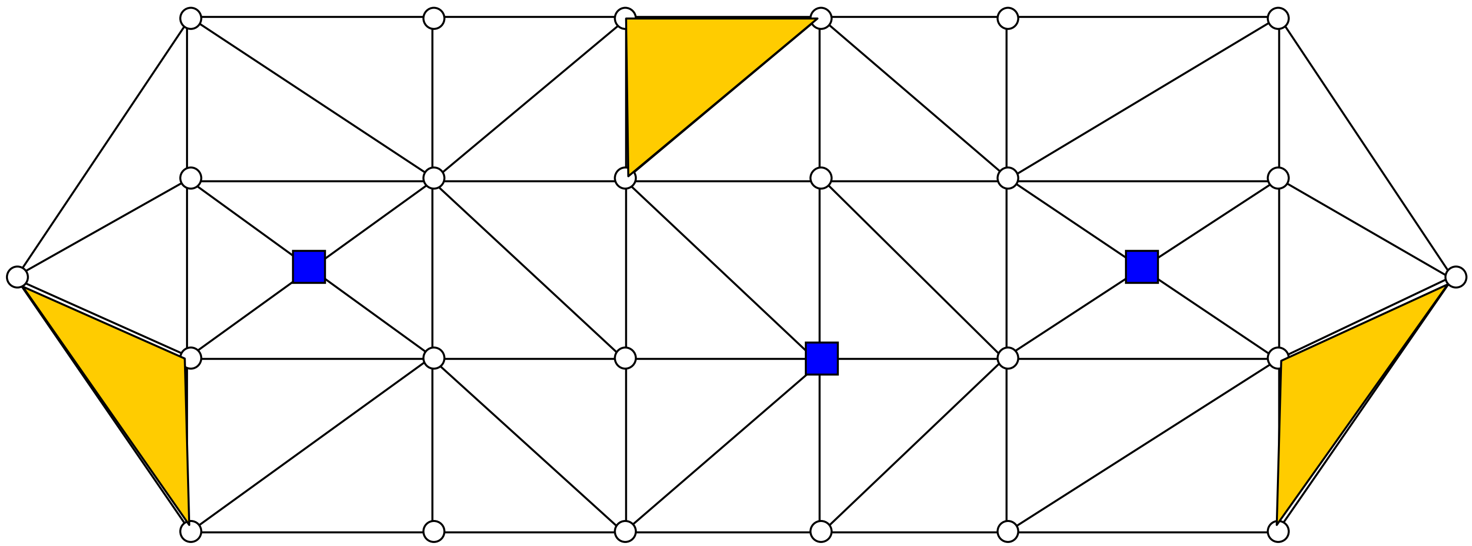
D = {■} is 2d-dominating set
is not 2d-guarding

$\gamma_{2d}(T) = 2$

REMOTE MONITORING (distance 2)

T=(V,E) triangulation graph

$\gamma_{2d}(T) < g_{2d}(T) < \beta_{2d}(T)$



G = {■} is 2d-guarding set

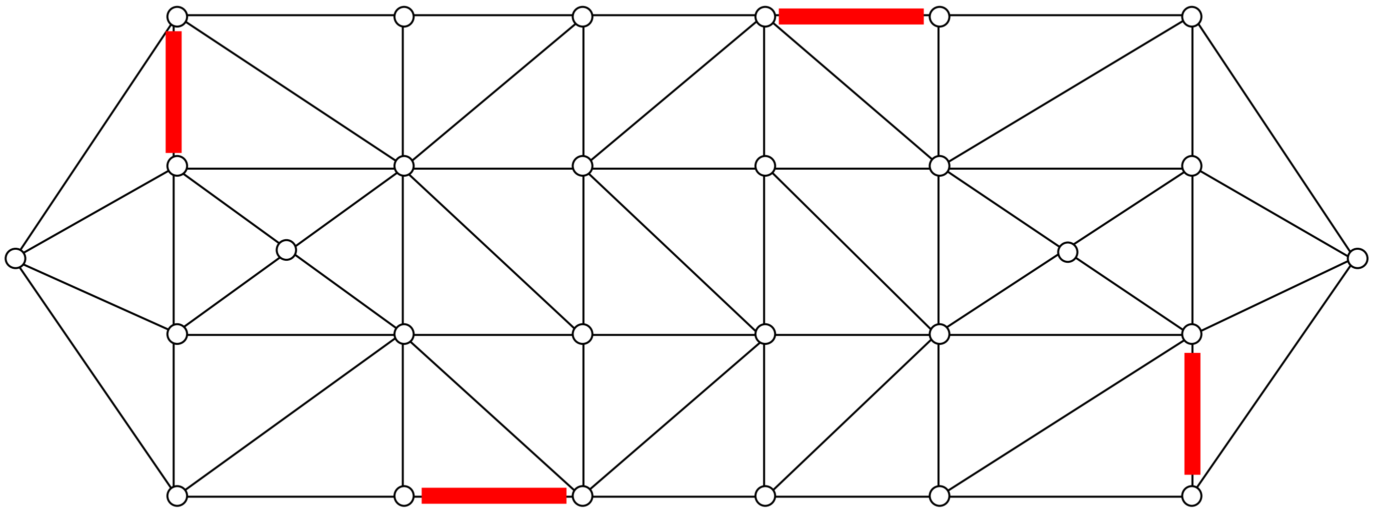
$g_{2d}(T) = 3$

Each yellow triangle needs a different guard

REMOTE MONITORING (distance 2)

T=(V,E) triangulation graph

$\gamma_{2d}(T) < g_{2d}(T) < \beta_{2d}(T)$



Each red edge needs a different vertex to be 2d-covered

$\beta_{2d}(T) > 4$

REMOTE MONITORING

T=(V,E) triangulation graph

$h_{kd}(n) = \max \{ h_{kd}(T) \ / \ T \text{ is a triangulation, } T = (V,E) \ , |V| = n\}$

Combinatorial bounds for $\gamma_{kd}(n)$, $g_{kd}(n)$, $\beta_{kd}(n)$

REMOTE MONITORING MOP's (distance 2)

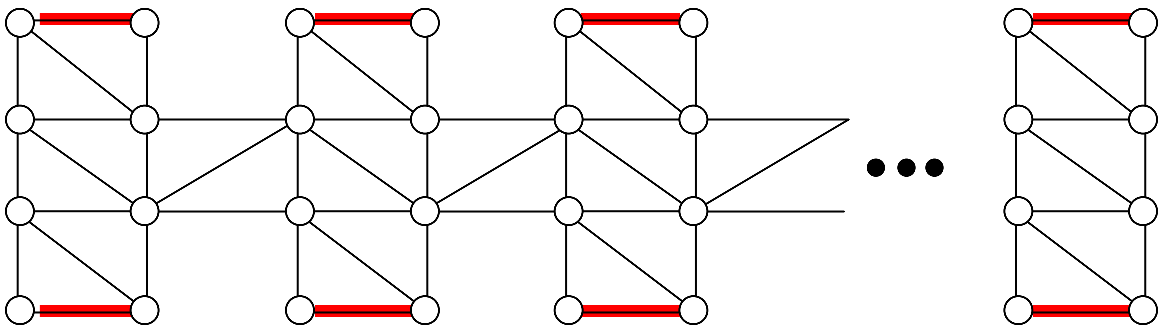
		Watched elements		
		Vertices	Faces	Edges
Watching from	Vertices	Dominating $\gamma_{2d}(n) = \left\lfloor \frac{n}{5} \right\rfloor$ (1)	Guarding $g_{2d}(n) = \left\lfloor \frac{n}{5} \right\rfloor$ (1)	Covering $\beta_{2d}(n) = \left\lfloor \frac{n}{4} \right\rfloor$ (1)
	Edges	**Edge-covering**	**Edge-guarding**	**Edge-dominating**
	Faces	**Face-vertex cover**	**Face-guarding**	**Face-edge cover**

(1) Canales, H., Martins, Matos, '13

VERTEX COVERING (MOP's, distance 2)

Every n-vertex maximal outerplanar graph, n≥4, can be 2d-covered with $\left\lfloor \frac{n}{4} \right\rfloor$ vertices and this bound is tight.

First, the lower bound

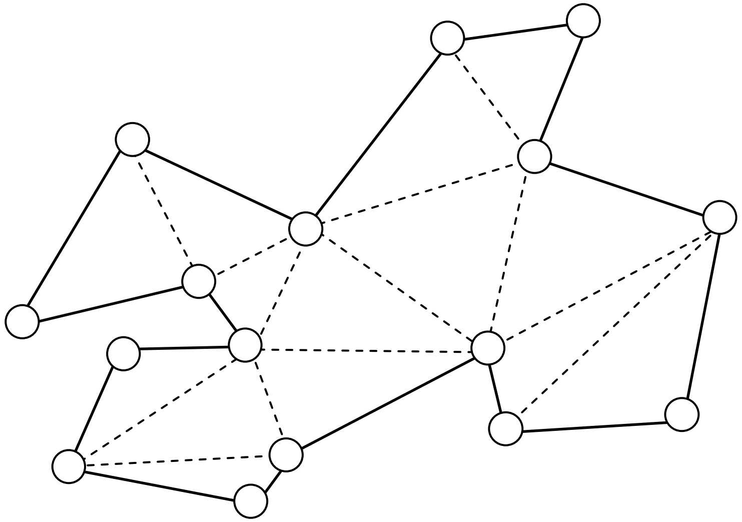


Red edges need different vertices to be 2d-covered, then

$$\left\lfloor \frac{n}{4} \right\rfloor \leq \beta'_{2d}(T)$$

VERTEX COVERING (MOP's, distance 2)

The edges of any T can be 2d-covered with $\left\lfloor \frac{n}{4} \right\rfloor$ vertices

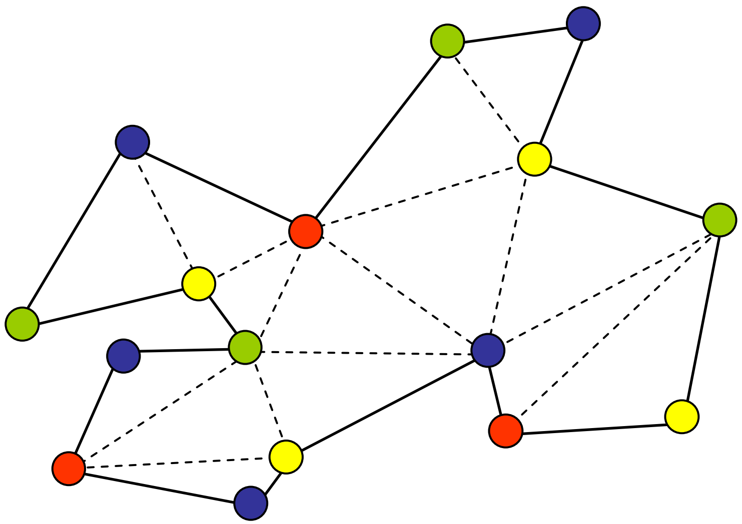


Lemma (Tokunaga '13)

The vertices of any n-MOP can be 4-colored such every 4-cycle has all 4 colors

VERTEX COVERING (MOP's, distance 2)

The edges of any T can be 2d-covered with $\left\lfloor \frac{n}{4} \right\rfloor$ vertices



Lemma (Tokunaga '13)

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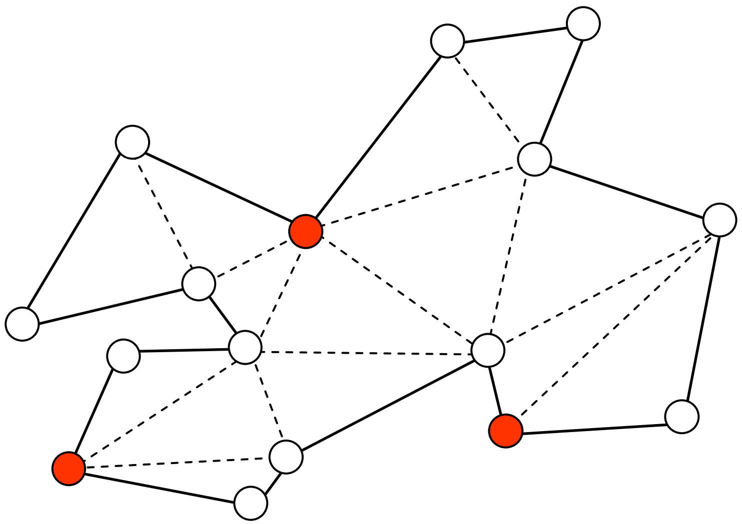
Vertices of same color are a 2d-vertex cover

VERTEX COVERING (MOP's, distance 2)

The edges of any T can be 2d-covered with $\left\lfloor \frac{n}{4} \right\rfloor$ vertices



Lemma (Tokunaga '13)

The vertices of any n-MOP can be 4-colored such every 4-cycle has all 4 colors

Vertices of same color are a 2d-vertex cover

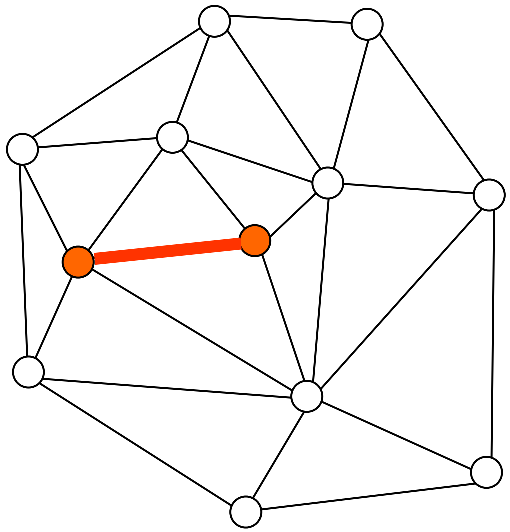
The vertices with the least used color are at most $\left\lfloor \frac{n}{4} \right\rfloor$

REMOTE MONITORING BY EDGES

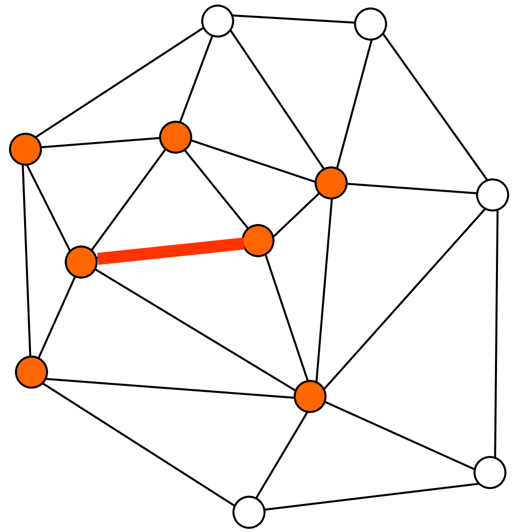
T=(V,E) triangulation graph

Edge-covering k-distance

An edge e kd-covers a vertex v if $dist_T(v, e) \leq k - 1$



k = 1 edge-covering

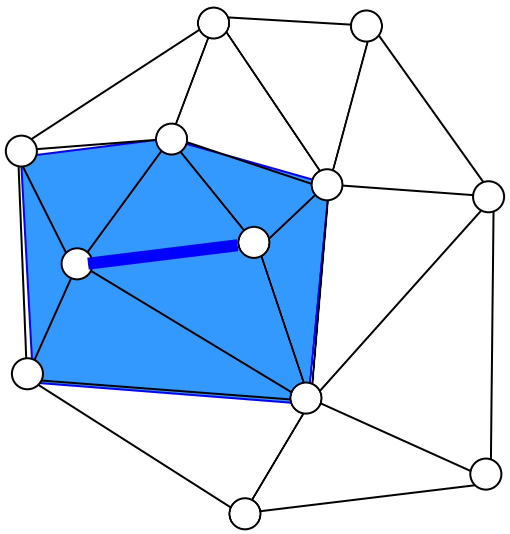


k = 2

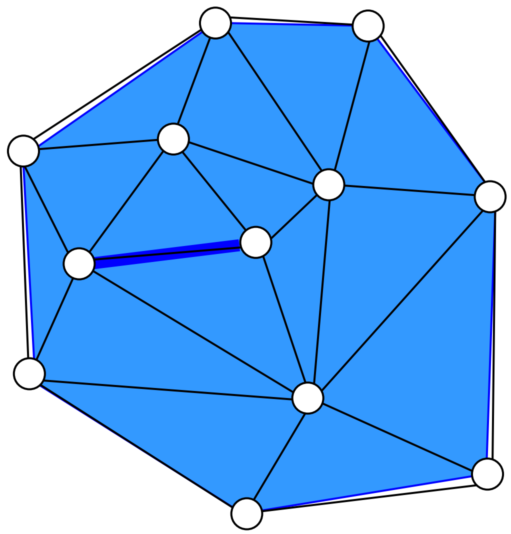
T=(V,E) triangulation graph

Edge-guarding k-distance

An edge e kd-guards a triangle T_i if $dist_T(T_i, e) \leq k - 1$



k = 1 edge-guarding



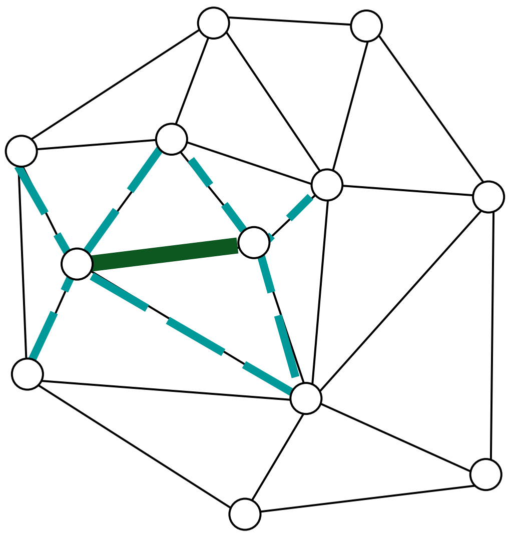
k = 2

REMOTE MONITORING BY EDGES

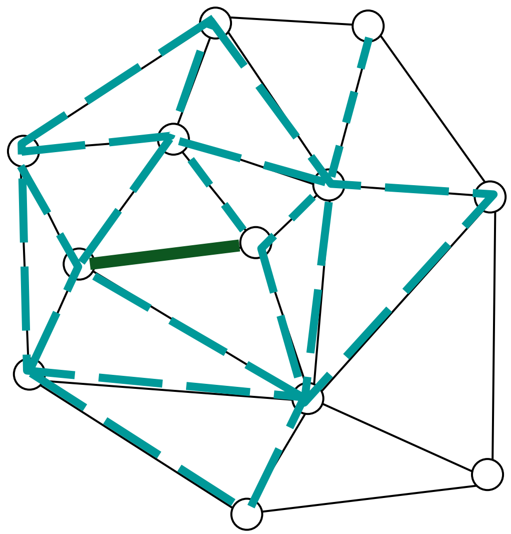
T=(V,E) triangulation graph

Edge-dominating k-distance

An edge e kd-dominates an edge e_i if $\text{dist}_T(e_i, e) \le k - 1$



k = 1 edge-domination



k = 2

REMOTE MONITORING MOP's (distance 2)

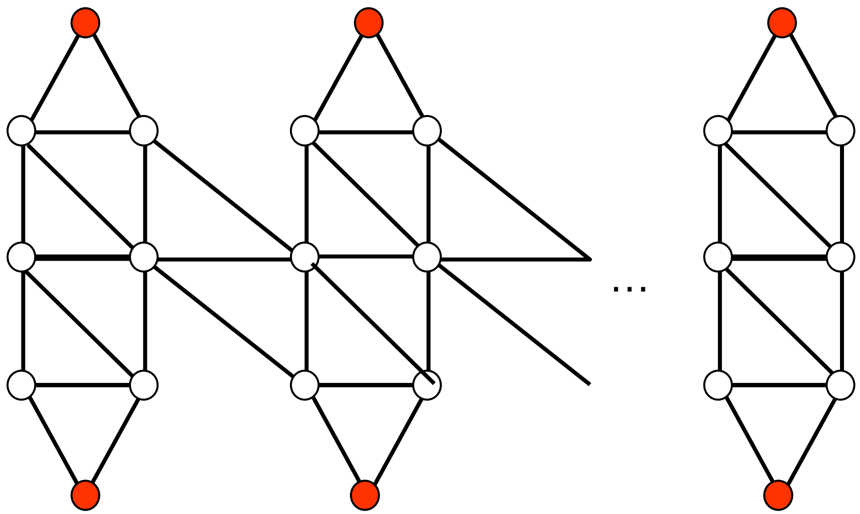
Watching from	Watched elements: Vertices	Watched elements: Faces	Watched elements: Edges
Vertices	Dominating $\gamma_{2d}(n) = \left\lfloor \frac{n}{5} \right\rfloor$	Guarding $g_{2d}(n) = \left\lfloor \frac{n}{5} \right\rfloor$	Covering $\beta_{2d}(n) = \left\lfloor \frac{n}{4} \right\rfloor$
Edges	**Edge-covering** $\beta'_{2d}(n) = \left\lfloor \frac{n}{4} \right\rfloor$	**Edge-guarding** $g^{e}_{2d}(n) = \left\lfloor \frac{n}{6} \right\rfloor$	**Edge-dominating** $\gamma'_{2d}(n) = \left\lfloor \frac{n}{5} \right\rfloor$
Faces	**Face-vertex cover** $f^{v}_{2d}(n) = \left\lfloor \frac{n}{4} \right\rfloor$	**Face-guarding** $g^{f}_{2d}(n) = \left\lfloor \frac{n}{6} \right\rfloor$	**Face-edge cover** $f^{e}_{2d}(n) = \left\lfloor \frac{n}{5} \right\rfloor$

H., Martins '14

EDGE COVERING (MOP's, distance 2)

Every n-vertex maximal outerplanar graph, n≥4, can be 2d-edge covered with $\left\lfloor \frac{n}{4} \right\rfloor$ edges and this bound is tight.

First, the lower bound



Red vertices need different edges to be 2d-edge-covered, then

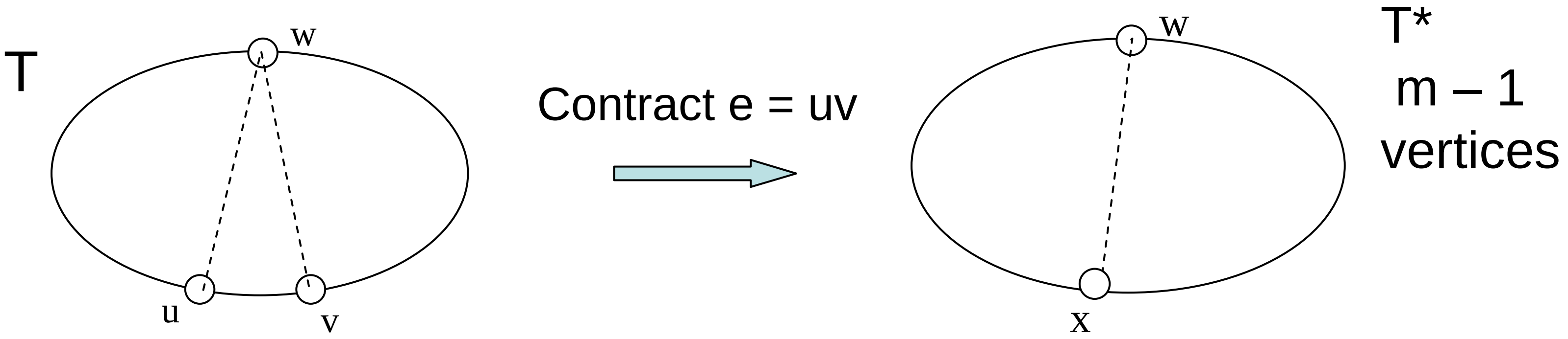
$$\left\lfloor \frac{n}{4} \right\rfloor \leq \beta'_{2d}(T)$$

EDGE COVERING (MOP's, distance 2)

Upper bound

Every n-vertex MOP T, with n ≥ 4, can be 2d-edge-covered by $\lfloor n/4 \rfloor$ edges

Lemma 3. Suppose that f(m) edges are always sufficient to guard any MOP T with m vertices. Let be e = uv an exterior edge of T. Then f(m−1) edges and an additional "collapsed edge" at the vertex u or v are sufficient to 2d-edge-cover T.



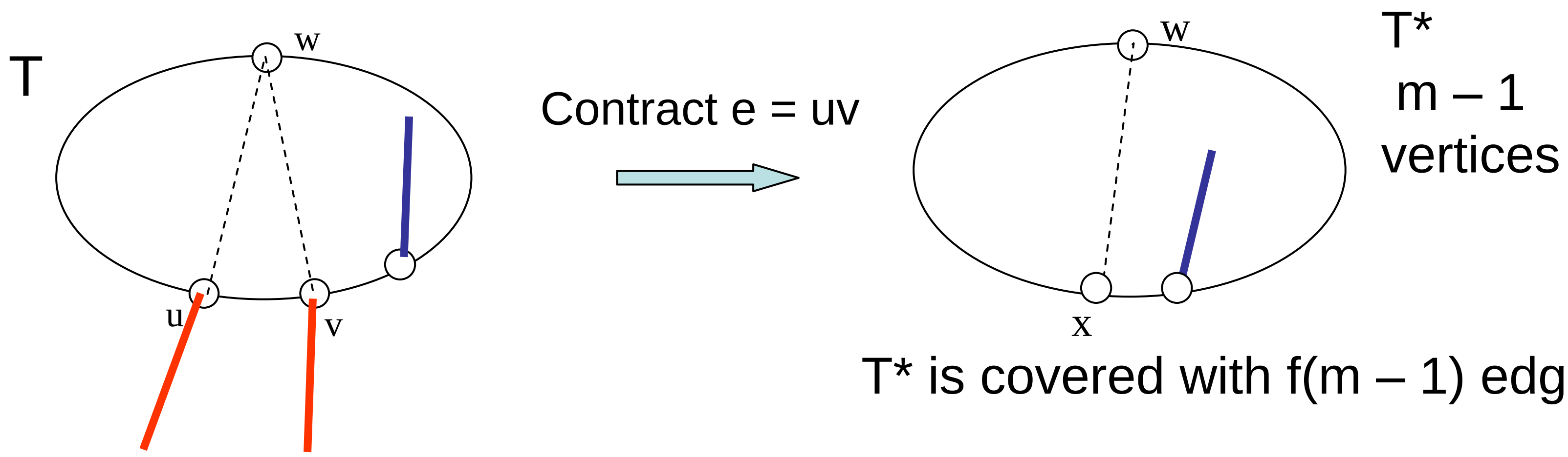
T* is covered with f(m – 1) edges

EDGE COVERING (MOP's, distance 2)

Upper bound

Every n-vertex MOP T, with $n \geq 4$, can be 2d-edge-covered by $\lfloor n/4 \rfloor$ edges

Lemma 3. Suppose that f(m) edges are always sufficient to guard any MOP T with m vertices. Let be e = uv an exterior edge of T. Then f(m−1) edges and an additional "collapsed edge" at the vertex u or v are sufficient to 2d-edge-cover T.



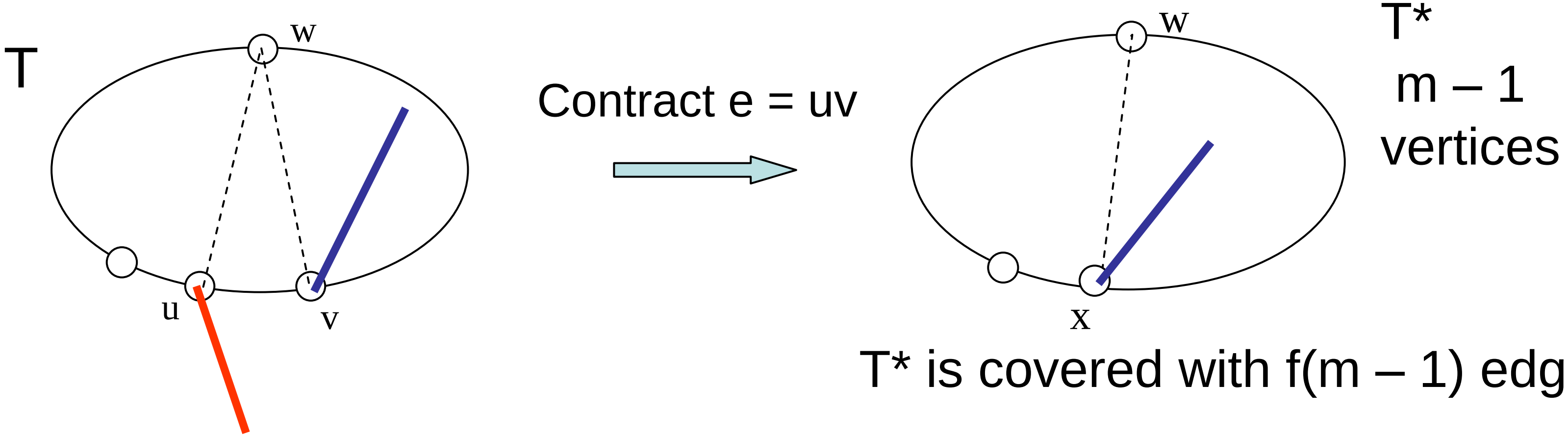
T* is covered with f(m – 1) edges

EDGE COVERING (MOP's, distance 2)

Upper bound

Every n-vertex MOP T, with n ≥ 4, can be 2d-edge-covered by $\lfloor n/4 \rfloor$ edges

Lemma 3. Suppose that f(m) edges are always sufficient to guard any MOP T with m vertices. Let be e = uv an exterior edge of T. Then f(m–1) edges and an additional "collapsed edge" at the vertex u or v are sufficient to 2d-edge-cover T.



T* is covered with f(m – 1) edges

EDGE COVERING (MOP's, distance 2)

Upper bound

Every n-vertex MOP T, with $n \geq 4$, can be 2d-edge-covered by $\lfloor n/4 \rfloor$ edges

Proof

Induction on n

Basic case: for $4 \leq n \leq 9$, easy

Inductive step: Let $n \geq 10$ and assume that the theorem holds for $n' < n$

Lemma 1 guarantees the existence of a diagonal that divides T in G_1 and G_2, such that G_1 has m = 5, 6, 7 or 8 exterior edges

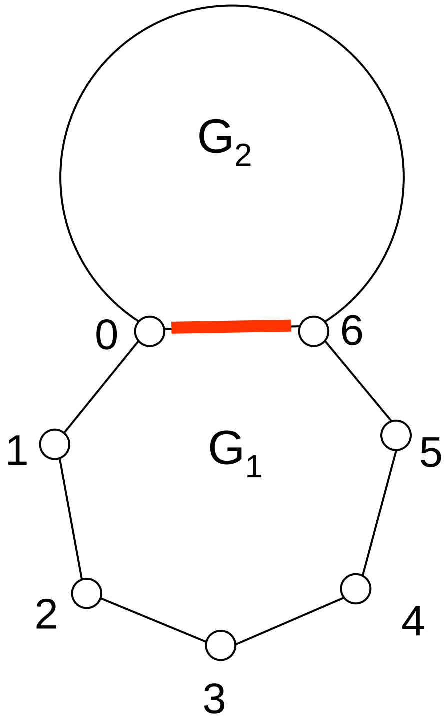
EDGE COVERING (MOP's, distance 2)

Upper bound

Every n-vertex MOP T, with $n \geq 4$, can be 2d-edge-covered by $\lfloor n/4 \rfloor$ edges

Proof

Case m = 6



G_2 has $n - 5$ exterior edges

⬇ I.H.

can be 2d-edge covered with
□(n−5)/4□ ≤ □n/4□−1
edges

G_1 has 7 exterior edges

⬇

can be 2d-edge covered with
one edge

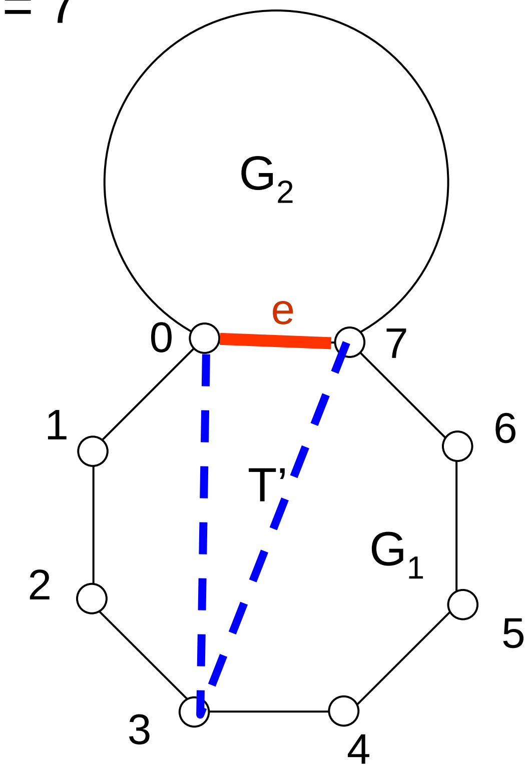
G can be 2d-edge covered by □n/4□ edges

EDGE COVERING (MOP's, distance 2)

Upper bound

Every n-vertex MOP T, with $n \geq 4$, can be 2d-edge-covered by $\lfloor n/4 \rfloor$ edges

Case m = 7



The presence of any of the internal edges (0,6), (0,5), (7,1) and (7,2) would violate the minimality of m

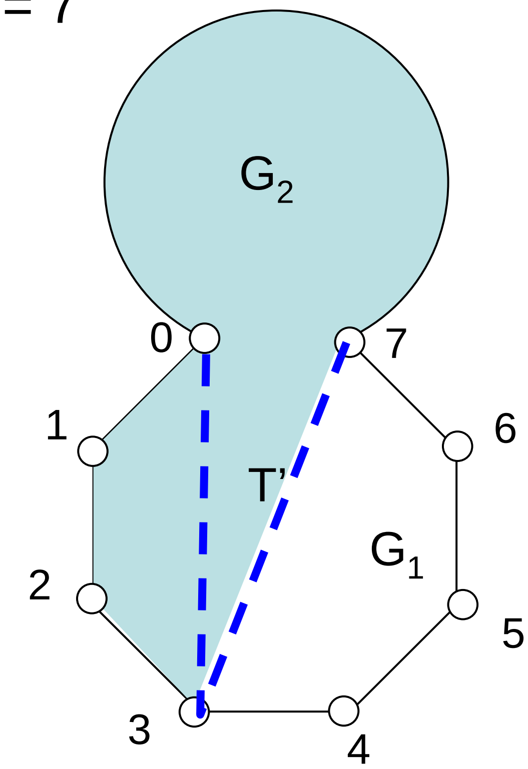
Thus, the triangle T' in G_1 that is bounded by e is (0,3,7) or (0,4,7)
We suppose that is (0,3,7)

EDGE COVERING (MOP's, distance 2)

Upper bound

Every n-vertex MOP T, with n ≥ 4, can be 2d-edge-covered by $\lfloor n/4 \rfloor$ edges

Case m = 7



Consider T* = G_2 + 01237
T* is maximal outerplanar graph and has n – 3 exterior edges

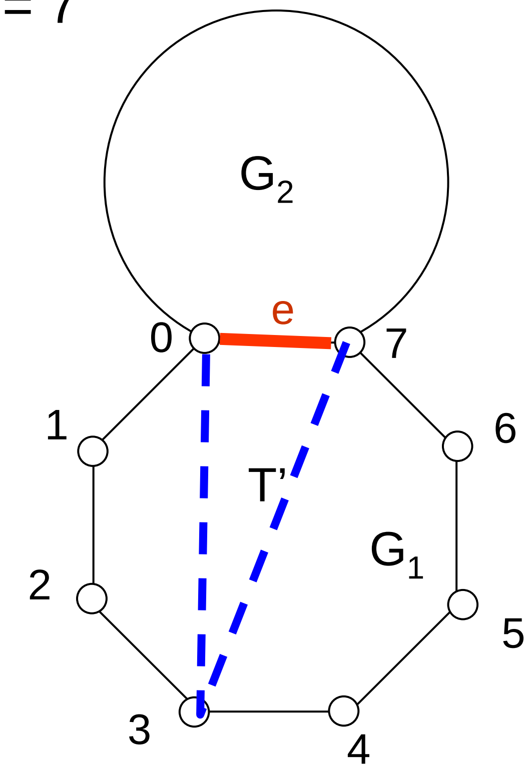
By lemma 3 T* can be 2d-edge covered with f(n – 4) = $\lfloor n/4 \rfloor - 1$ edges, and an additional "collapsed edge" at the vertex 3 or 7.

EDGE COVERING (MOP's, distance 2)

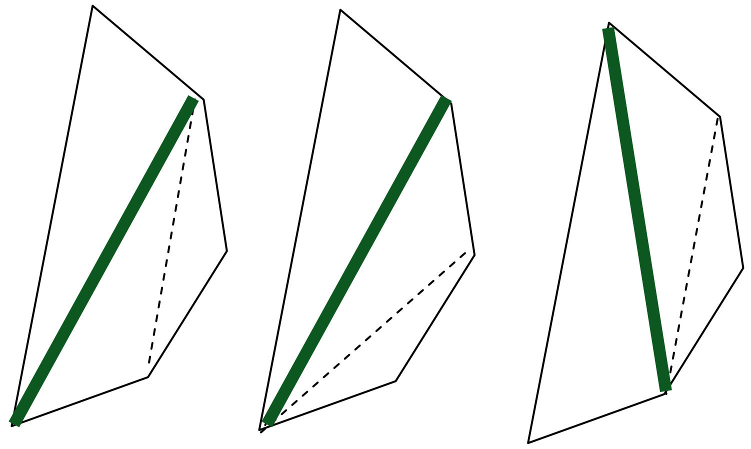
Upper bound

Every n-vertex MOP T, with n ≥ 4, can be 2d-edge-covered by $\lfloor n/4 \rfloor$ edges

Case m = 7



The "collapsed edge" at 3 or 7, also 2d-edge-covers the pentagon 34567, regardless how it is triangulated



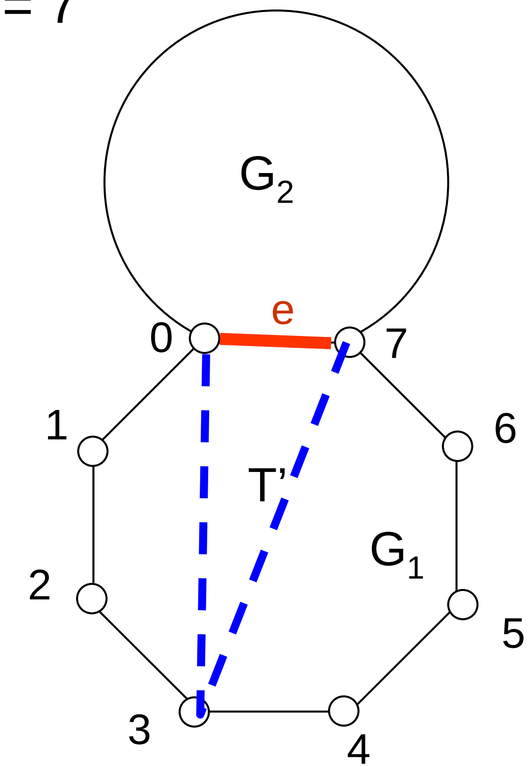
Therefore, T is 2d-edge covered by $\left\lfloor \frac{n}{4} \right\rfloor$ edges

EDGE COVERING (MOP's, distance 2)

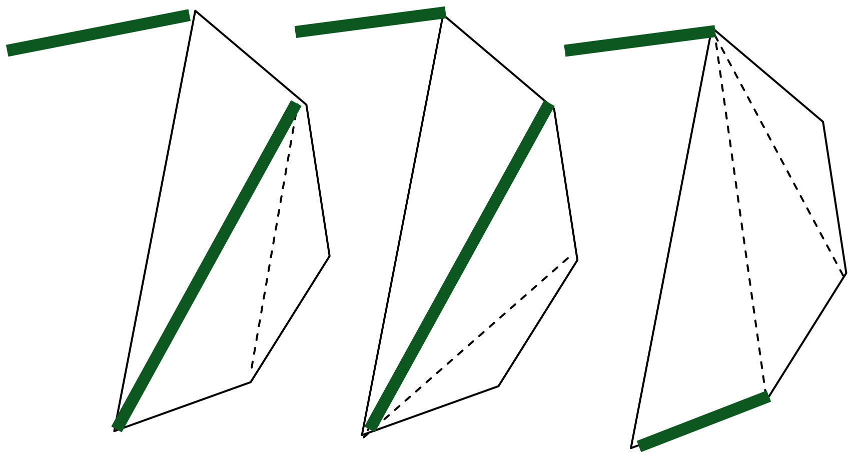
Upper bound

Every n-vertex MOP T, with n ≥ 4, can be 2d-edge-covered by $\lfloor n/4 \rfloor$ edges

Case m = 7



The "collapsed edge" at 3 or 7, also 2d-edge-covers the pentagon 34567, regardless how it is triangulated



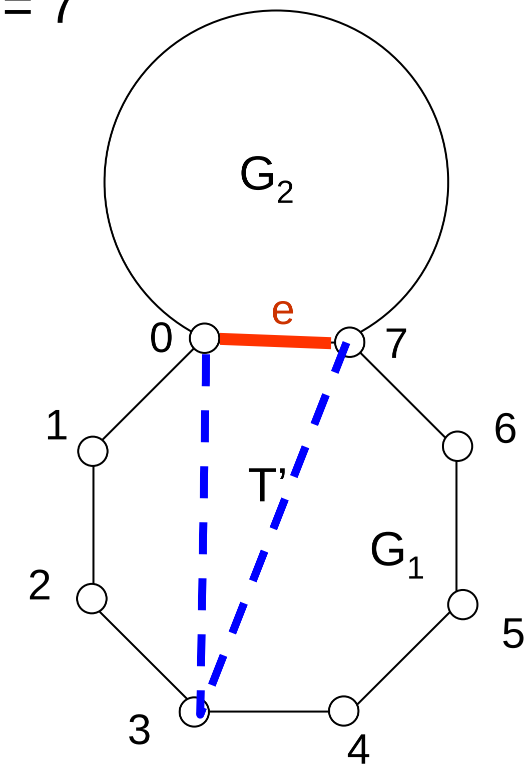
Therefore, T is 2d-edge covered by $\left\lfloor \frac{n}{4} \right\rfloor$ edges

EDGE COVERING (MOP's, distance 2)

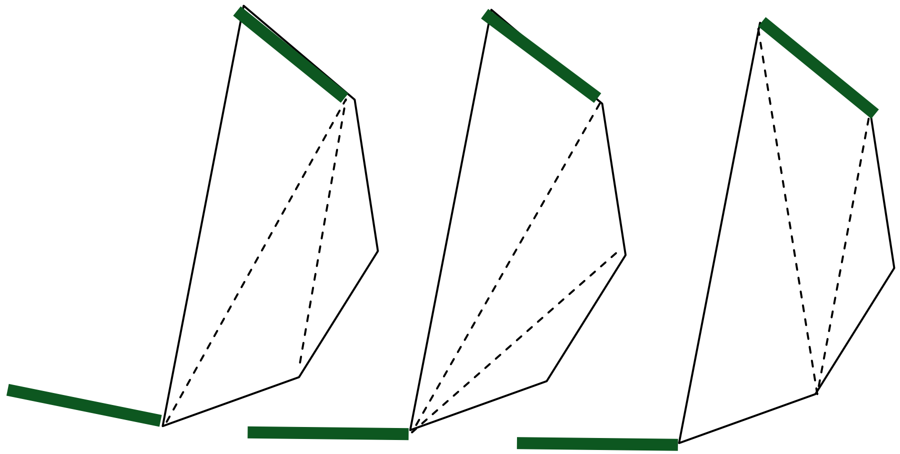
Upper bound

Every n-vertex MOP T, with $n \geq 4$, can be 2d-edge-covered by $\lfloor n/4 \rfloor$ edges

Case m = 7



The "collapsed edge" at 3 or 7, also 2d-edge-covers the pentagon 34567, regardless how it is triangulated

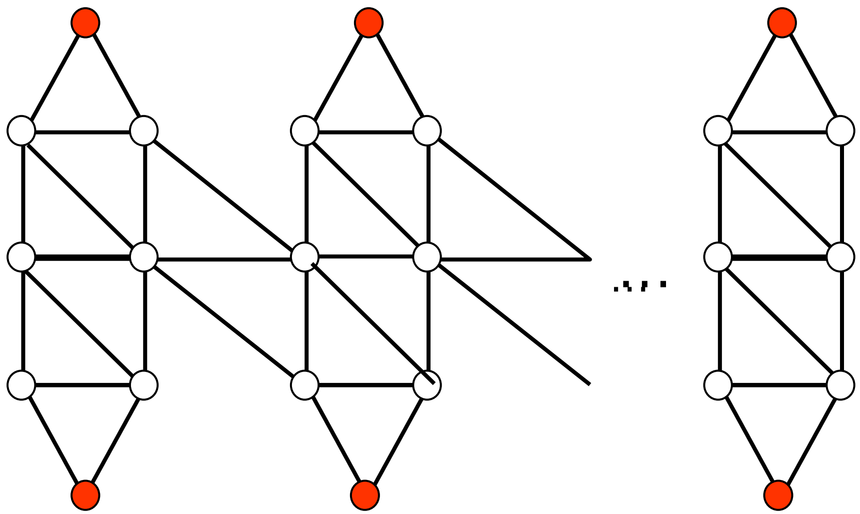


Therefore, T is 2d-edge covered by $\left\lfloor \frac{n}{4} \right\rfloor$ edges

FACE-VERTEX COVERING (MOP's, distance 2)

Every n-vertex maximal outerplanar graph, n≥4, can be 2d-face-vertex covered with $\left\lfloor \frac{n}{4} \right\rfloor$ faces and this bound is tight.

First, the lower bound



Red vertices need different faces to be 2d-face-vertex-covered, then

$$\left\lfloor \frac{n}{4} \right\rfloor \leq f_{2d}^{v}(T)$$

FACE-VERTEX COVERING (MOP's, distance 2)

Every n-vertex maximal outerplanar graph, n≥4, can be 2d-face-vertex covered with $\left\lfloor \frac{n}{4} \right\rfloor$ faces and this bound is tight.

If T is a maximal outerplanar graph then

$$f_{2d}^{v}(T) \leq \beta'_{2d}(T)$$

Therefore,

$$\left\lfloor \frac{n}{4} \right\rfloor \leq f_{2d}^{v}(n) \leq \beta'_{2d}(n) \leq \left\lfloor \frac{n}{4} \right\rfloor$$

REMOTE MONITORING MOP's (distance k)

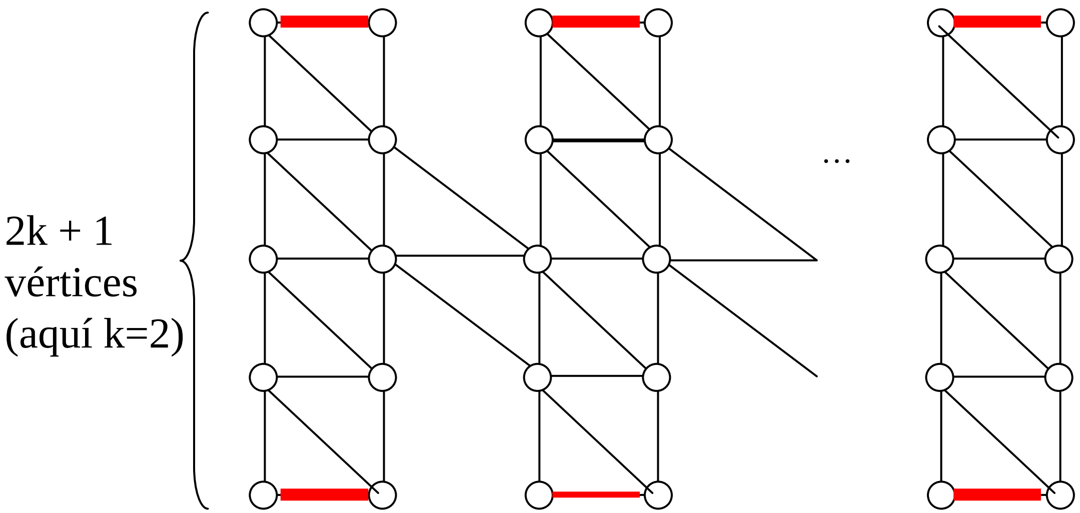
Watching from \ Watched elements	Vertices	Faces	Edges
Vertices	Dominating $\gamma_{kd}(n) = \left\lfloor \frac{n}{2k+1} \right\rfloor$	Guarding $g_{2d}(n) = \left\lfloor \frac{n}{2k+1} \right\rfloor$	Covering $\beta_{2d}(n) = \left\lfloor \frac{n}{2k} \right\rfloor$
Edges	**Edge-covering** $\beta'_{2d}(n) = \left\lfloor \frac{n}{2k} \right\rfloor$	**Edge-guarding** $g^{e}_{2d}(n) = \left\lfloor \frac{n}{2k+2} \right\rfloor$	**Edge-dominating** $\gamma'_{2d}(n) = \left\lfloor \frac{n}{2k+1} \right\rfloor$
Faces	**Face-vertex cover** $f^{v}_{2d}(n) = \left\lfloor \frac{n}{2k} \right\rfloor$	**Face-guarding** $g^{f}_{2d}(n) = \left\lfloor \frac{n}{2k+2} \right\rfloor$	**Face-edge cover** $f^{e}_{2d}(n) = \left\lfloor \frac{n}{2k+1} \right\rfloor$

H., Martins '15

EDGE DOMINATING (MOP's, distance k)

Every n-vertex maximal outerplanar graph, $n \geq 2k + 1$, can be kd-edge dominated by $\left\lfloor \frac{n}{2k+1} \right\rfloor$ edges and this bound is tight.

First, the lower bound



Red edges need different edges to be kd-dominated, then

$$\left\lfloor \frac{n}{2k+1} \right\rfloor \leq \gamma'_{kd}(T)$$

EDGE DOMINATING (MOP's, distance k)

Upper bound

Every n-vertex MOP T, n≥2k+1, can be 2d-dominated by $\lfloor n/(2k+1) \rfloor$ edges,

that is

$$\gamma'_{kd}(n) \leq \left\lfloor \frac{n}{2k+1} \right\rfloor$$

Lemma 1. $\gamma'_{kd}(n) \leq \gamma'_{kd}(n+1)$

Lemma 2. $\gamma'_{kd}(n) = 1$ if $3 \leq n \leq 4k+1$

Lemma 3. $\gamma'_{kd}(n) = 2$ if $n = 4k+2$ or $n = 4k+3$

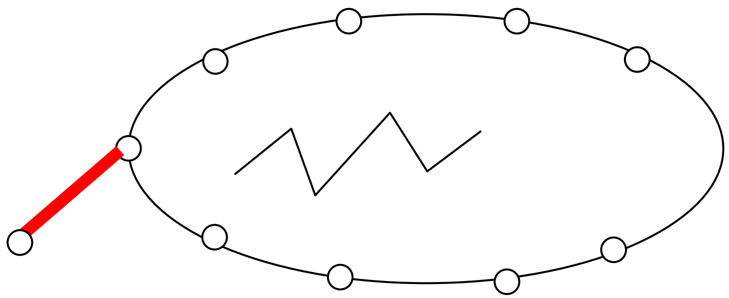
Lemma 4. (Contraction) Suppose that f(m) edges kd-dominate all the edges of any MOP T with m vertices. Let be e = uv an exterior edge of T. Then f(m−1) edges and an additional "collapsed edge" at the vertex u or v are sufficient to kd-edge-dominate T.

EDGE DOMINATING (MOP's, distance k)

Lemma 2. $\gamma'_{kd}(n) = 1$ **if** $3 \leq n \leq 4k + 1$

Case $n \leq 2k$

Any collapsed edge at any vertex of dominates all the edges



k = 5

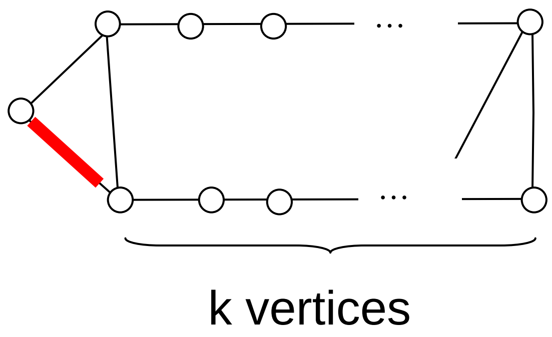
Lemma 2. $\gamma'_{kd}(n) = 1$ **if** $3 \leq n \leq 4k + 1$

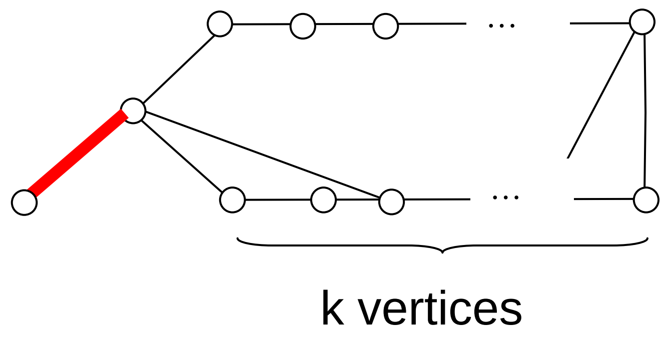
Case $n = 2k + 1$

Any edge dominates all the edges

OR

Any collapsed edge at vertex of degree > 2 dominates all the edges



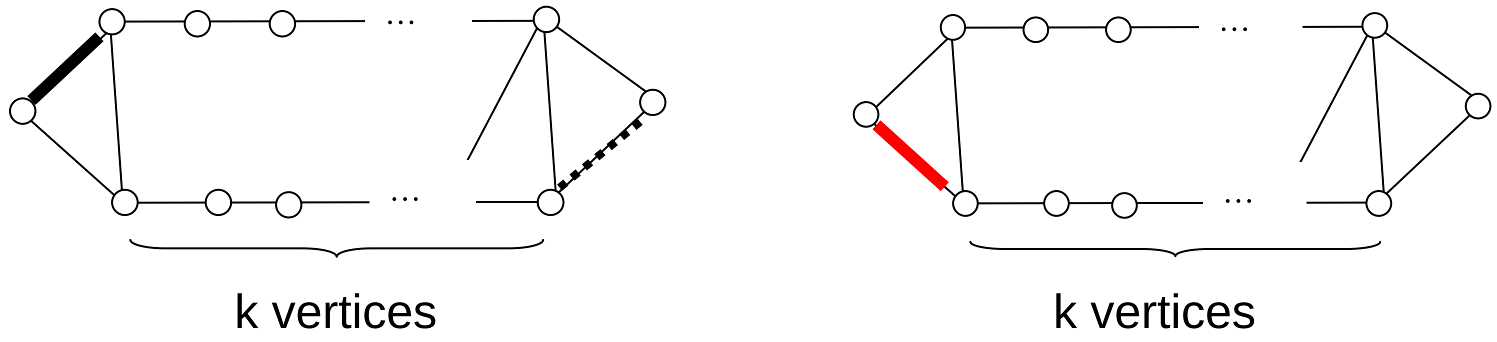


Lemma 2. $\gamma'_{kd}(n) = 1$ if $3 \leq n \leq 4k + 1$

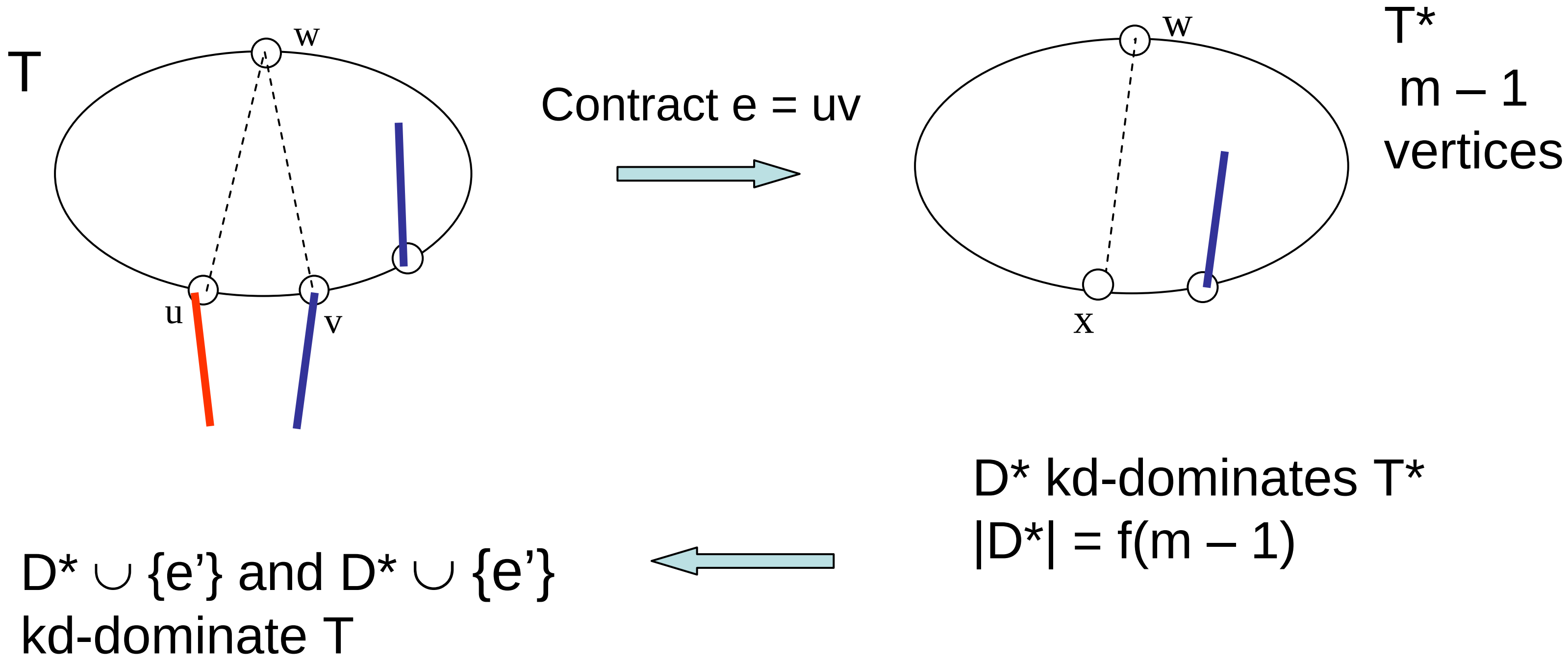
Case $n = 2k + 2$

Subcase A) Any interior edge (both extremes degree ≥ 3) dominates all the edges

Subcase B) One of the two incident edges in a vertex of degree 2 dominates all the edges

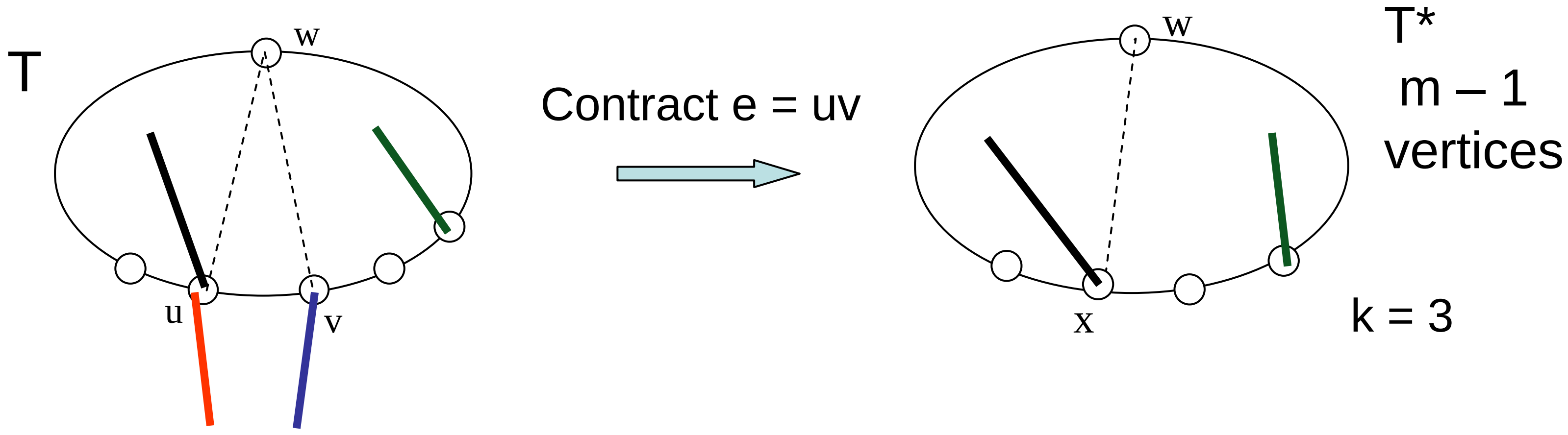


Lemma 4. (Contraction) Suppose that f(m) edges kd-dominate all the edges of any MOP T with m vertices. Let be e = uv an exterior edge of T. Then f(m−1) edges and an additional "collapsed edge" at the vertex u or v are sufficient to kd-edge-dominate T.



EDGE DOMINATING (MOP's, distance k)

Lemma 4. (Contraction) Suppose that f(m) edges kd-dominate all the edges of any MOP T with m vertices. Let be e = uv an exterior edge of T. Then f(m–1) edges and an additional "collapsed edge" at the vertex u or v are sufficient to kd-edge-dominate T.

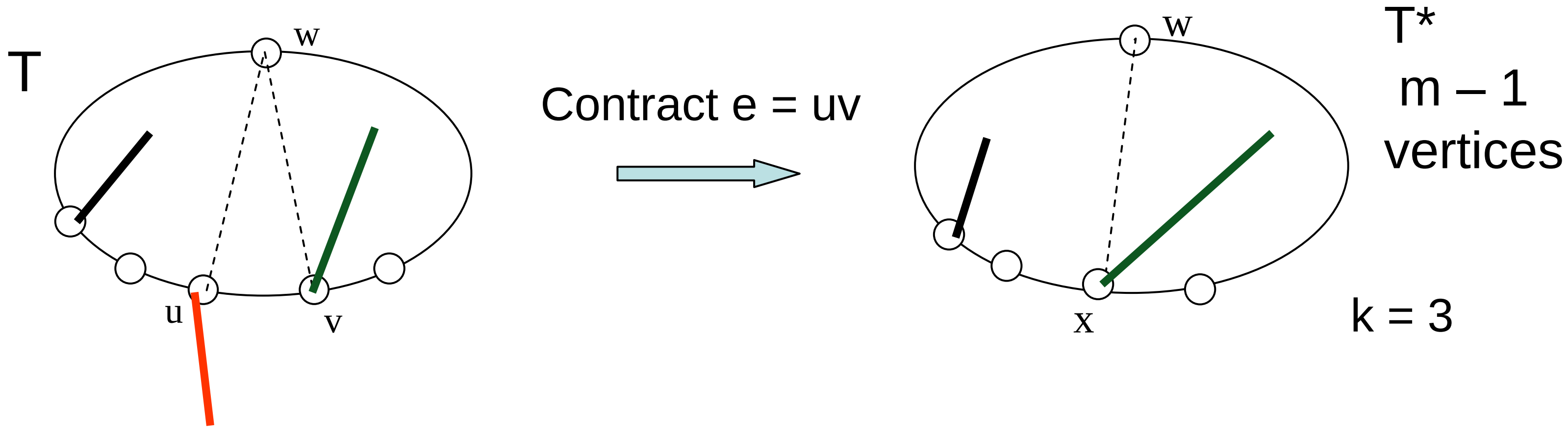


D* 3d-dominates T*
|D*| = f(m – 1)

D* ∪ {e'} and D* ∪ {e''}
3d-dominate T

EDGE DOMINATING (MOP's, distance k)

Lemma 4. (Contraction) Suppose that f(m) edges kd-dominate all the edges of any MOP T with m vertices. Let be e = uv an exterior edge of T. Then f(m−1) edges and an additional "collapsed edge" at the vertex u or v are sufficient to kd-edge-dominate T.



D* ∪ {e'}
3d-dominate T

D* 3d-dominates T*
|D*| = f(m – 1)

EDGE DOMINATING (MOP's, distance k)

Upper bound

Every n-vertex MOP T, $n \geq 2k+1$, can be 2d-dominated by $\lfloor n/(2k+1) \rfloor$ edges,

Proof

Induction on n

Basic case: for $3 \leq n \leq 4k + 3$, lemmas 2, 3

Inductive step: Let $n \geq 4k + 4$ and assume that the theorem holds for $n' < n$

Lemma 1 guarantees the existence of a diagonal that divides T in G_1 and G_2, such that G_1 has m exterior edges, $2k + 2 \leq m \leq 4k + 2$

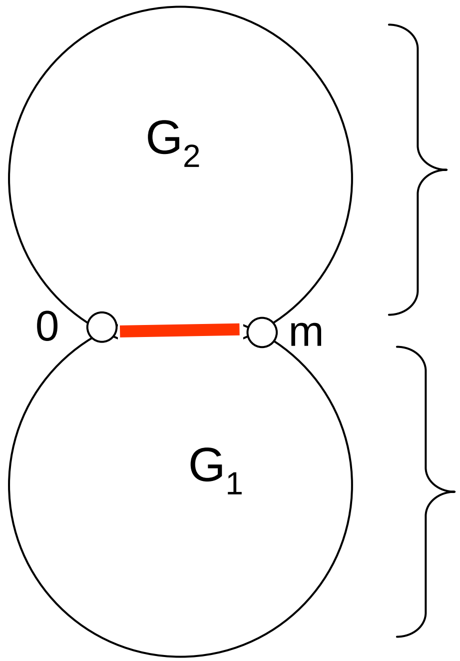
EDGE DOMINATING (MOP's, distance k)

Upper bound

Every n-vertex MOP T, $n \geq 2k+1$, can be 2d-dominated by $\lfloor n/(2k+1) \rfloor$ edges,

Proof

Case $m \leq 4k$



G_2 has $n - m + 1 \leq n - 2k - 1$ vertices

⬇ I.H.

is 2d-dominated by $\left\lfloor \frac{n-2k-1}{2k+1} \right\rfloor = \left\lfloor \frac{n}{2k+1} \right\rfloor - 1$ edges

G_1 has $m + 1 \leq 4k + 1$ vertices

⬇ Lemma 2

can be 2d-dominated by one edge

T can be 2d-dominated by $\lfloor n/(2k+1) \rfloor$ edges

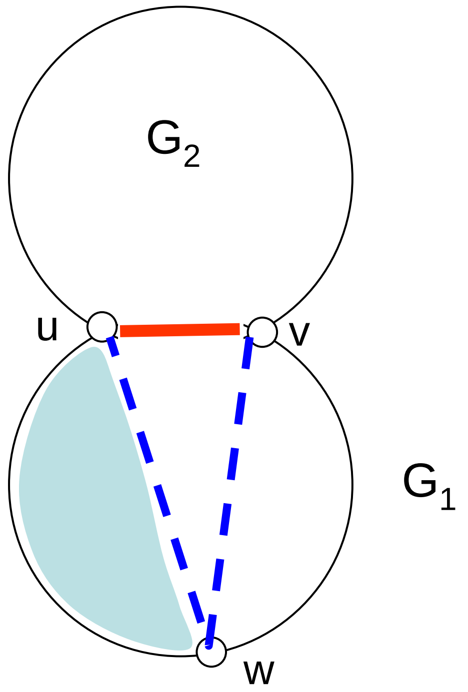
EDGE DOMINATING (MOP's, distance k)

Upper bound

Every n-vertex MOP T, n≥2k+1, can be 2d-dominated by $\lfloor n/(2k+1) \rfloor$ edges,

Case m = 4k + 1

G_1 has m + 1 = 4k +2 vertices



w apex of triangle T* in G_1 that is bounded by e

C exterior cycle of G_1 $dist_C(u,v)= 4k + 1$

By minimality of m ≥ 2k + 2

$dist_C(u,w)= 2k + 1$, $dist_C(w,v)= 2k$

T' triangulation determined by uw and C

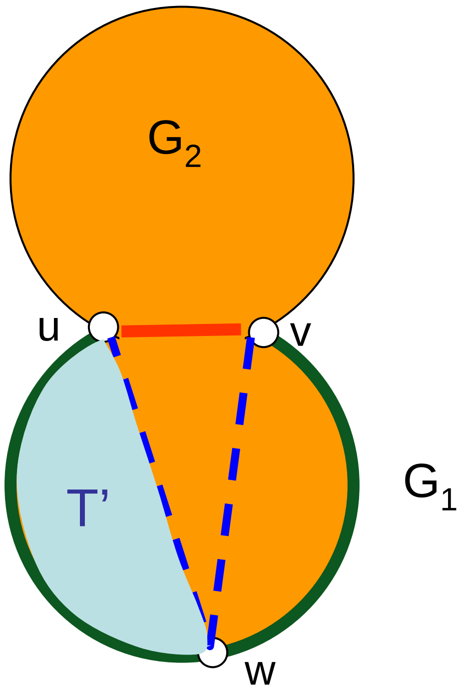
T'' = ($G_2 \cup G_1 \setminus$ T')

Upper bound

Every n-vertex MOP T, $n \geq 2k+1$, can be 2d-dominated by $\lfloor n/(2k+1) \rfloor$ edges,

Case $m = 4k + 1$

G_1 has $m + 1 = 4k + 2$ vertices



w apex of triangle T* in G_1 that is bounded by e

C exterior cycle of G_1 $dist_C(u,v) = 4k + 1$

By minimality of $m \geq 2k + 2$

$dist_C(u,w) = 2k + 1$, $dist_C(w,v) = 2k$

T' triangulation determined by uw and C

$T'' = (G_2 \cup G_1 \setminus T')$

T' $2k + 2$ vertices

T'' $n – (2k + 1) + 1 = n – 2k$ vertices

EDGE DOMINATING (MOP's, distance k)

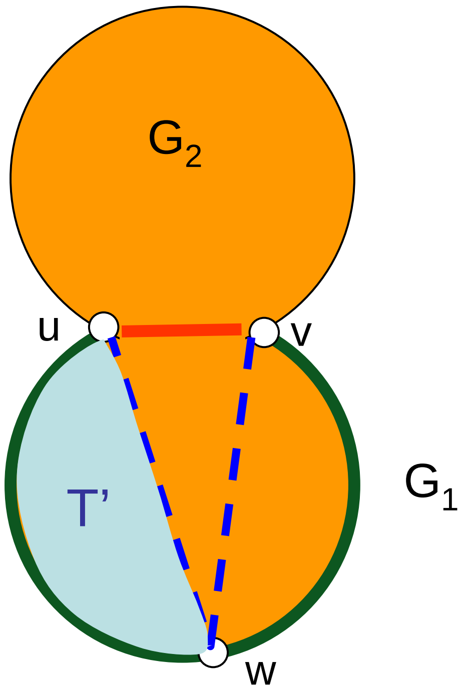
Upper bound

Every n-vertex MOP T, n≥2k+1, can be 2d-dominated by $\lfloor n/(2k+1) \rfloor$ edges,

Case m = 4k + 1

T' 2k + 2 vertices

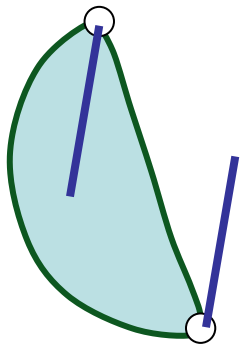
T'' n – (2k + 1) + 1 = n – 2k vertices



By lemma 4 T'' can be 2d-edge dominated with

$$f(n-2k-1)=\left\lfloor \frac{n}{2k+1} \right\rfloor -1$$ edges and an

additional "collapsed edge" (*) at the vertex u or w.



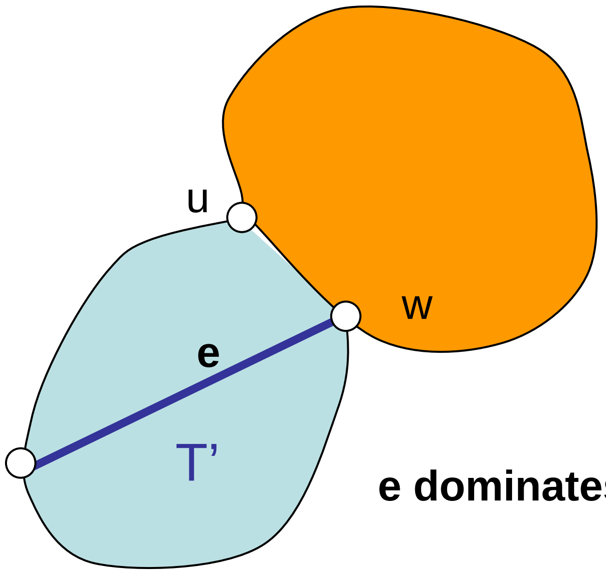
These edges dominate all edges of T'

EDGE DOMINATING (MOP's, distance k)

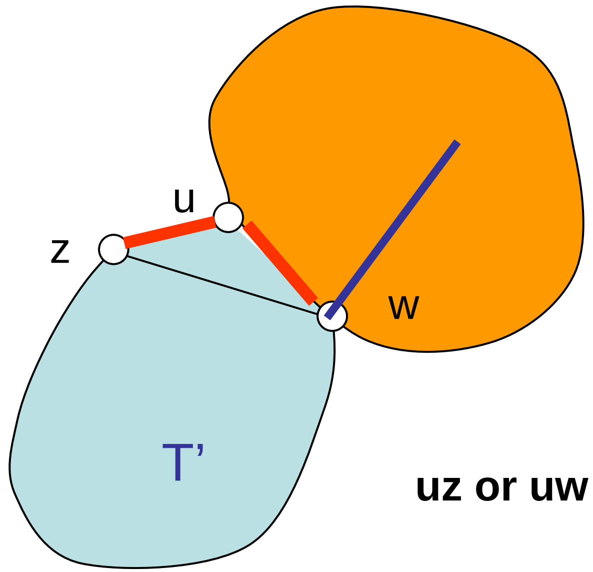
Upper bound

Every n-vertex MOP T, n≥2k+1, can be 2d-dominated by $\lfloor n/(2k+1) \rfloor$ edges,

Case m = 4k + 1   T' 2k + 2 vertices



e dominates T'



uz or uw dominate T'

If we can choose collapsed edge with extremes degree ≥ 3

If we can not …

EDGE DOMINATING (MOP's, distance k)

Every n-vertex maximal outerplanar graph, $n \geq 2k + 1$, can be 2d-edge dominated by $\lfloor n/(2k+1) \rfloor$ edges. And this bound is tight in the worst case, that is

$$\gamma'_{kd}(n) = \left\lfloor \frac{n}{2k+1} \right\rfloor$$

REMOTE MONITORING TRIANGULATION (distance 2)

Watching from		Watched elements: Vertices	Watched elements: Faces	Watched elements: Edges
	Vertices	Dominating + Guarding $\left\lfloor \frac{n}{5} \right\rfloor \leq \gamma_{2d}(n) \leq g_{2d}(n) \leq \left\lfloor \frac{n}{4} \right\rfloor$		**Covering** $\left\lfloor \frac{n}{4} \right\rfloor \leq \beta_{2d}(n) \leq \left\lfloor \frac{n}{3} \right\rfloor$
	Edges	**Edge-covering** $\left\lfloor \frac{n}{4} \right\rfloor \leq \beta'_{2d}(n) \leq \left\lfloor \frac{n}{3} \right\rfloor$	Edge-guarding	**Edge-dominating**
	Faces	**Face-vertex cover**	**Face-guarding** ?	**Face-edge cover** ?

Taller GC, (Abellanas, Canales, H. Martins, Orden, Ramos) marzo 2014

- Plane graphs (TRIANGULATIONS).
  Control by vertices, edges or faces

- **REMOTE** domination, covering, guarding, …

- Combinatorial bounds for
  MAXIMAL OUTERPLANAR GRAPHS
  TRIANGULATIONS (partial results)

- **FUTURE WORK: Triangulations**
  more parameters of domination

Thanks for your attention!!