



Universidad Politécnica
de Madrid

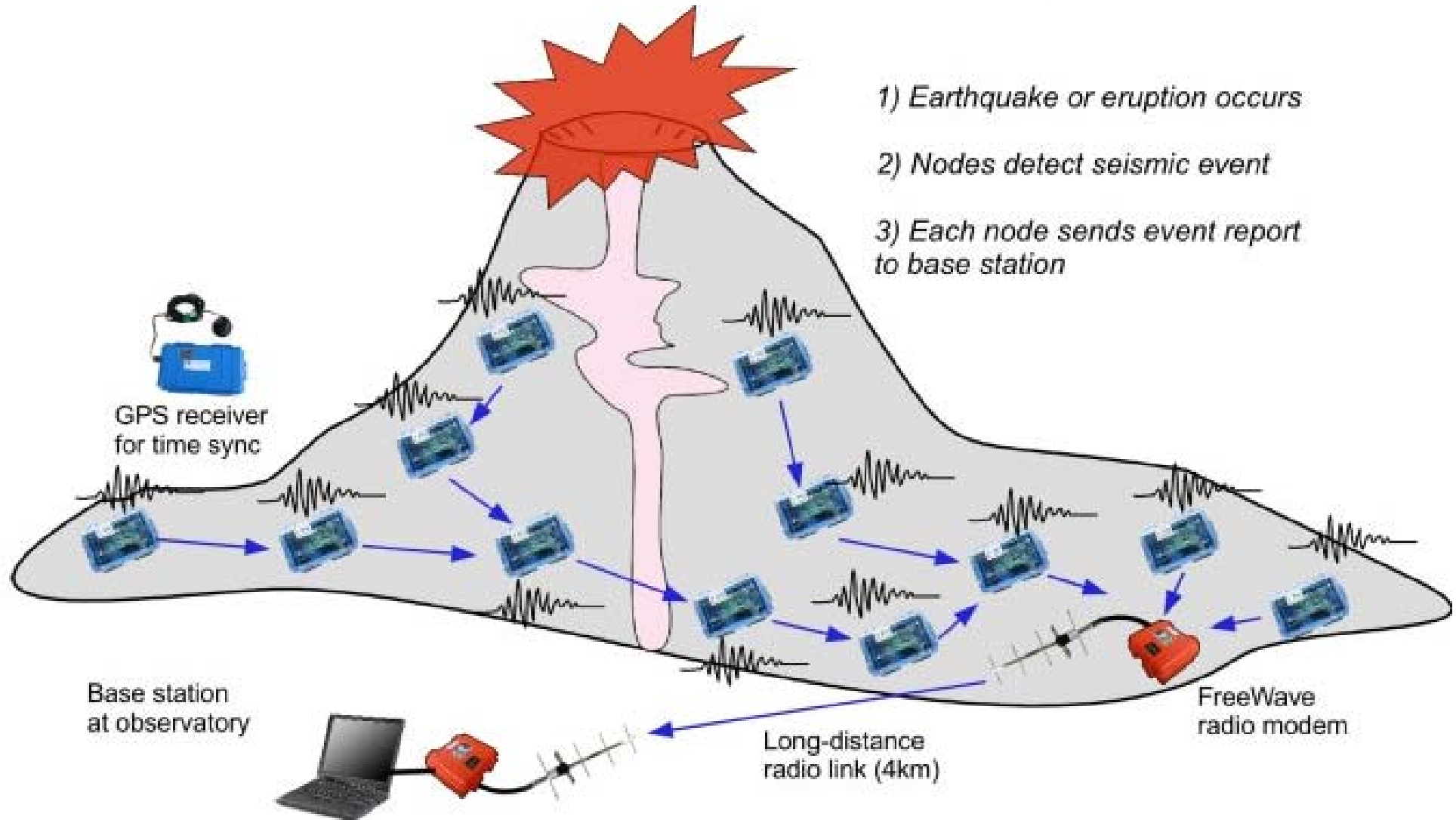
Geometric networks

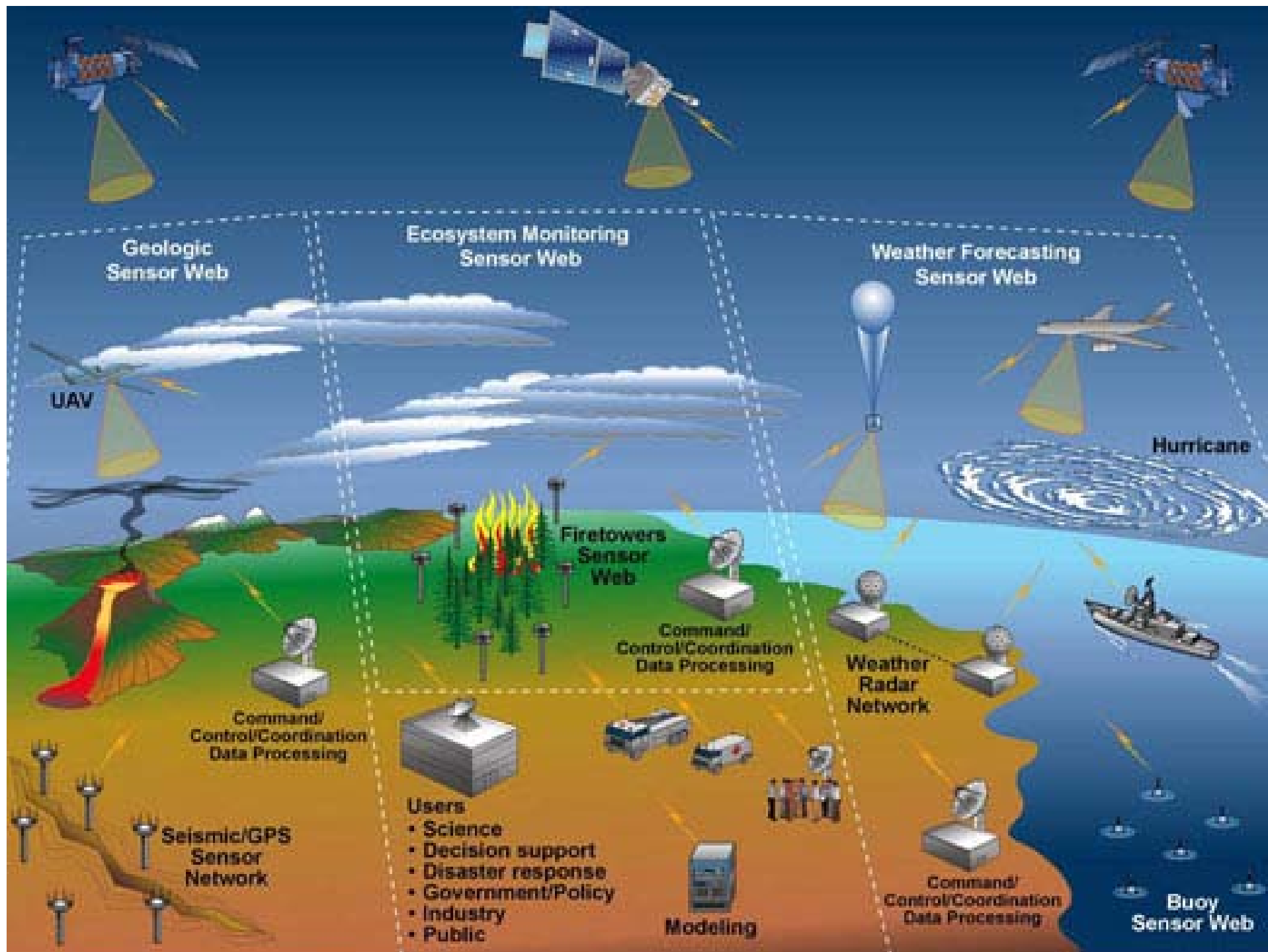
Global problems, local solutions

Gregorio Hernández
UPM

UPC Computational Geometry Seminar, November 17, 2010

Wireless networks





...other networks



T	Tram	M	Metro	R	Rodalies
L1	Hospital de Bellvitge	L9	Can Zam	R1	Molins de Rei
L2	Badalona	L10	La Sagrera	R2	Castelldefels
L3	Zona Universitària	L11	Trinitat Nova	R3	L'Hospitalet
L4	Trinitat Nova	L12	Can Cuiàs	R4	St. Vicenç
L5	La Pau	L13	Funicular de Montjuïc	R5	Maresme
L6	Cornel·la Centre	L14	Can Zam	R6	Can Sabades
L7	Molí Nou - Ciutat Cooperativa	L15	Trinitat Nova	R7	Can Masnou
L8	Sant Joan de Vilatorrada	L16	Trinitat Nova	R8	Can Masnou
L9	Can Zam	L17	Trinitat Nova	R9	Can Masnou
L10	La Sagrera	L18	Trinitat Nova	R10	Can Masnou
L11	Trinitat Nova	L19	Trinitat Nova	R11	Can Masnou
L12	Can Cuiàs	L20	Trinitat Nova	R12	Can Masnou
L13	Funicular de Montjuïc	L21	Trinitat Nova	R13	Can Masnou
L14	Can Zam	L22	Trinitat Nova	R14	Can Masnou
L15	Trinitat Nova	L23	Trinitat Nova	R15	Can Masnou
L16	Trinitat Nova	L24	Trinitat Nova	R16	Can Masnou
L17	Trinitat Nova	L25	Trinitat Nova	R17	Can Masnou
L18	Trinitat Nova	L26	Trinitat Nova	R18	Can Masnou
L19	Trinitat Nova	L27	Trinitat Nova	R19	Can Masnou
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L28	Trinitat Nova	L36	Trinitat Nova	R28	Can Masnou
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L50	Trinitat Nova	L58	Trinitat Nova	R50	Can Masnou

FGC	Rodalies	Tram
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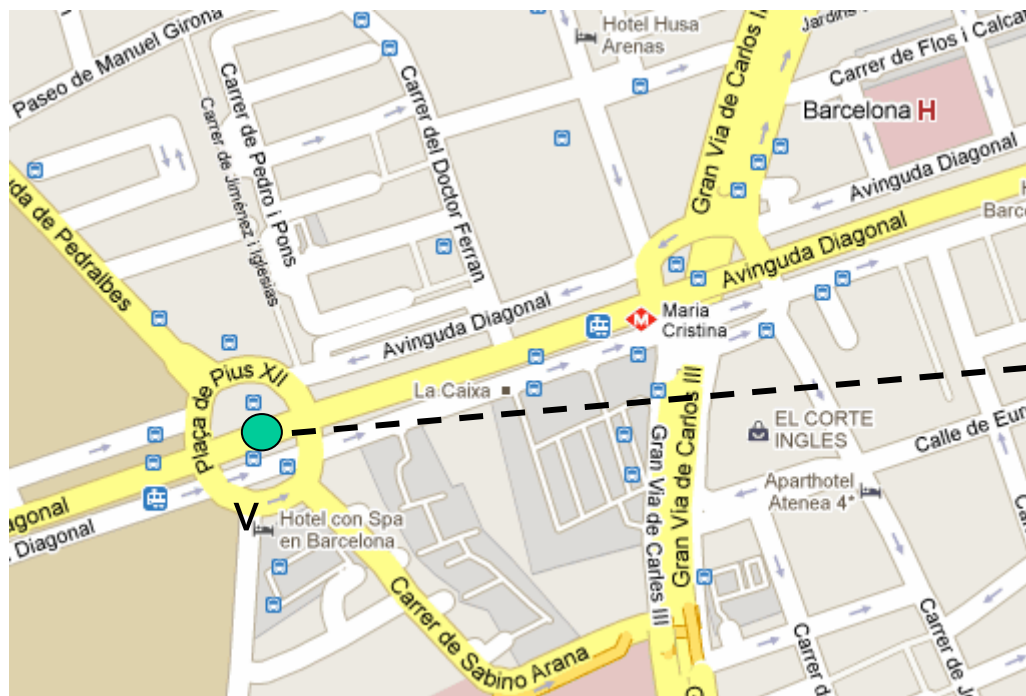
FGC	Rodalies	Tram
L1	R1	T1
L2	R2	T2
L3	R3	T3
L4	R4	T4
L5	R5	T5
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Metro

...other networks

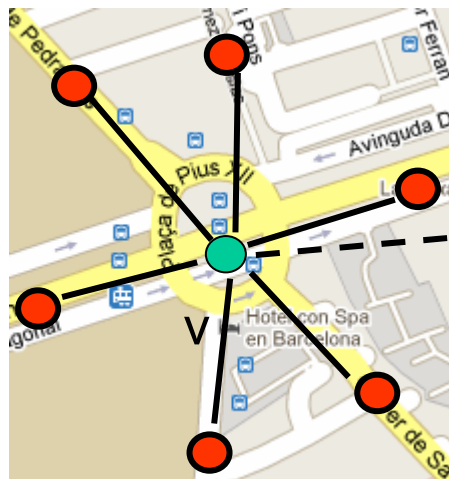
A tourist in Barcelona



Sagrada Família

...other networks

A tourist in Barcelona



Sagrada Família

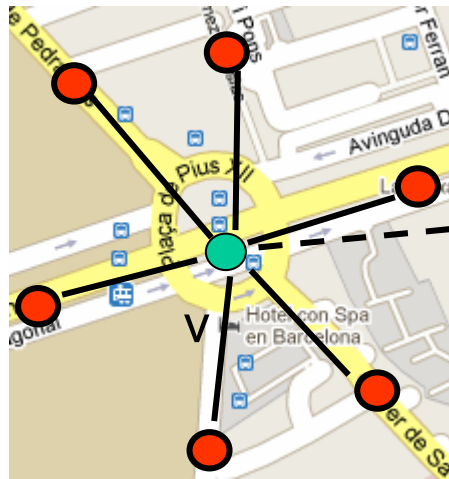
NO MAP

Local information (coordinates of v , target, and neighbors $N(v)$)

Limited memory allocation

Ecologically sound algorithms

...other networks



Sagrada Família

How can we move in
an unknown network?

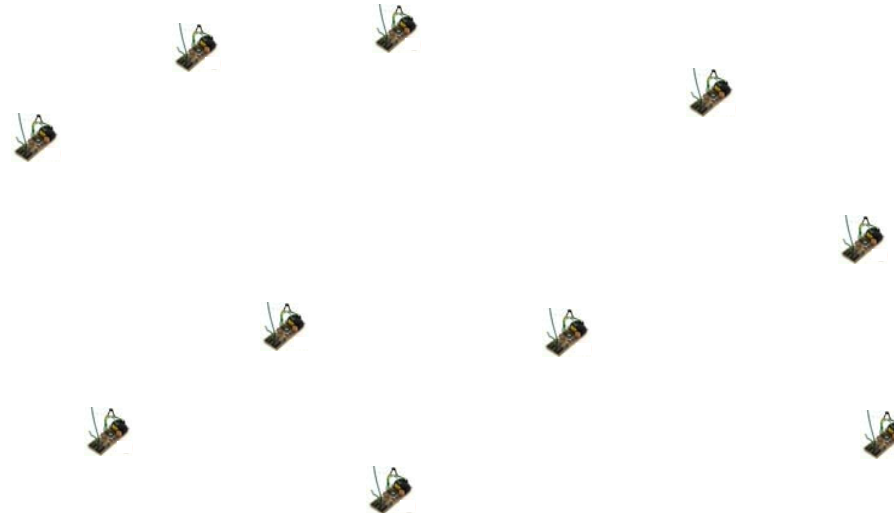
Wireless networks

With the sensors ...



No map!

No paths!



Wireless networks

- (1) How to organize the network?**
- (2) How to send messages?**
- (3) How to recover, store and index the data of the network?**

Wireless networks

(1) How to organize the network? **DESIGN**

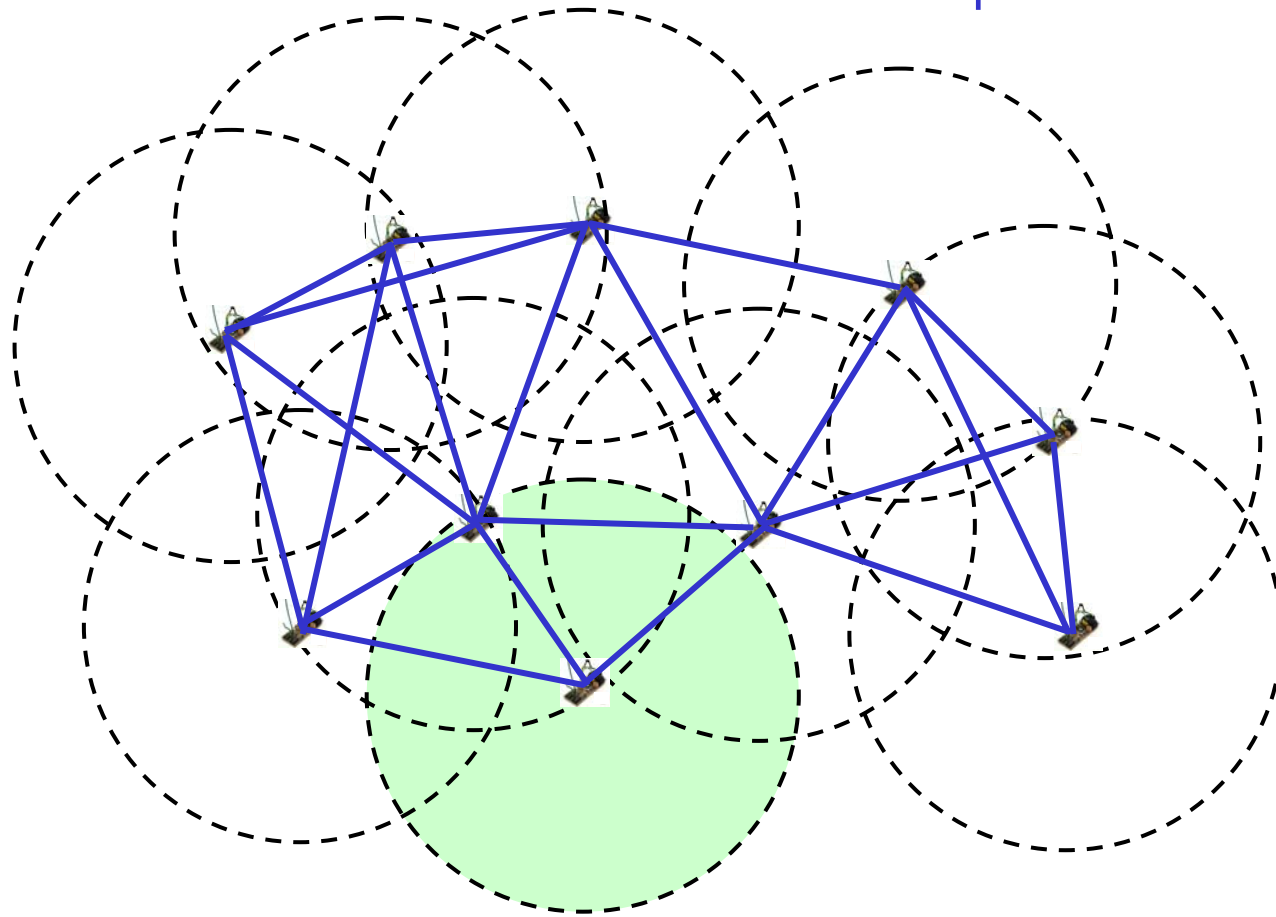
(2) How to send messages? **ROUTING**

Wireless networks

With the sensors ...

No map!

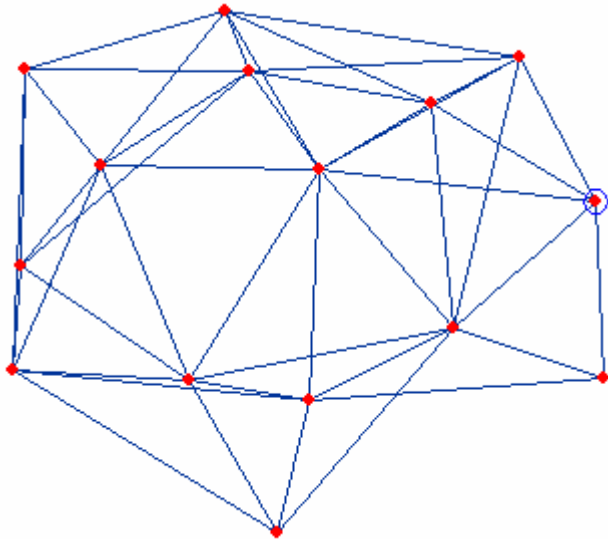
No paths!



Wireless networks

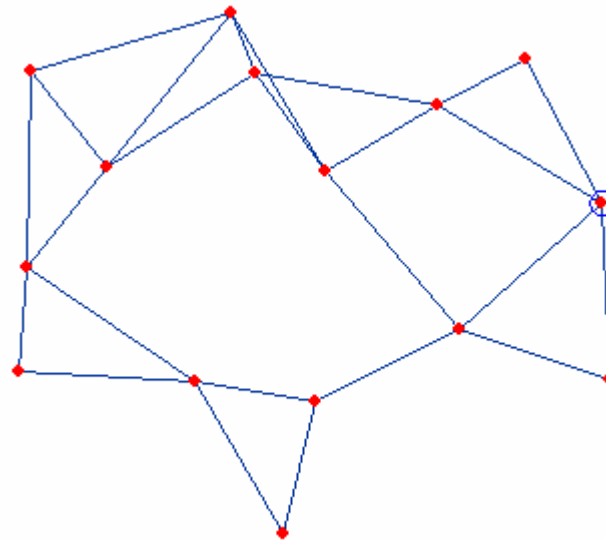
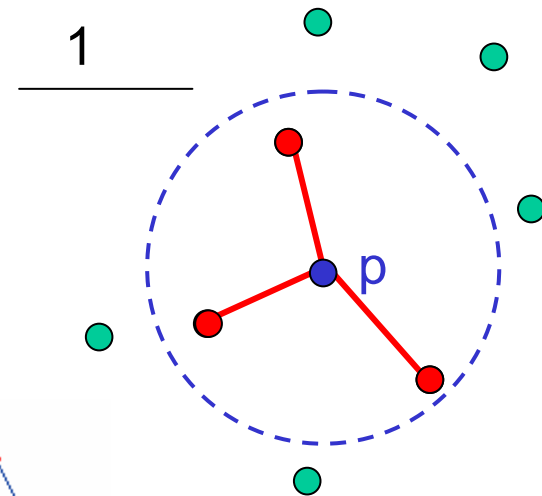
NETWORK MODEL

Neighbors of p are the points of S contained in the circle of center p and radius 1



Unit Disk Graph

UDG



Wireless networks

LOCAL ALGORITHM

For each node we know:

- (1) Position of u
- (2) Neighbors of u (until distance k)
- (3) The subyacent graph is $UDG(S)$

Wireless networks

ROUTING PROBLEM

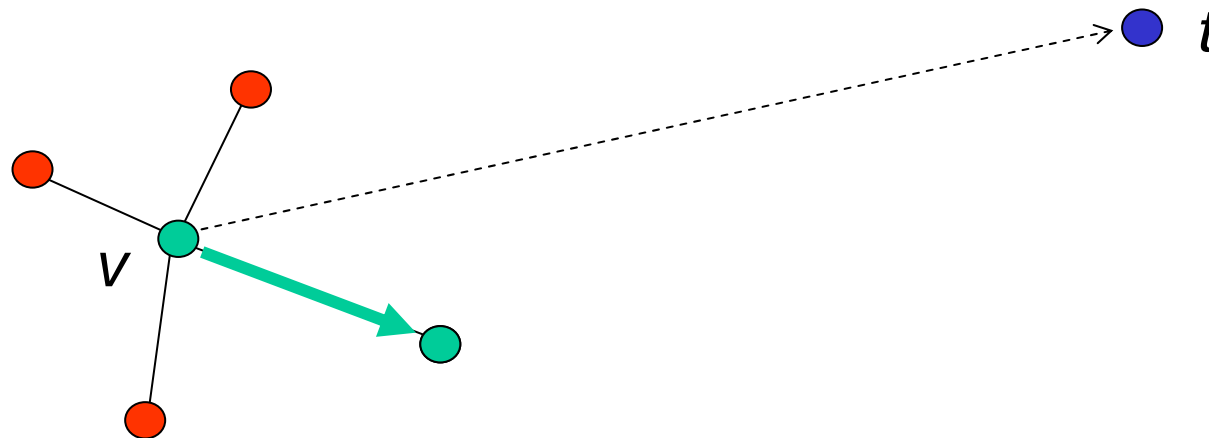
PROBLEM 1

Let G be a geometric planar network. Is there a **deterministic algorithm** that allows an agent A standing at a vertex s to travel to a vertex t of G under the following conditions:

- (1) A has a **constant amount of memory**; that is, at any point in time A knows the position of s and t , and the positions of a constant number of nodes in G .
- (2) When the agent visits a vertex x of G , it can use the list of vertices (and their positions) adjacent to x
- (3) A is not allowed to leave any marks along its way

Wireless networks ROUTING

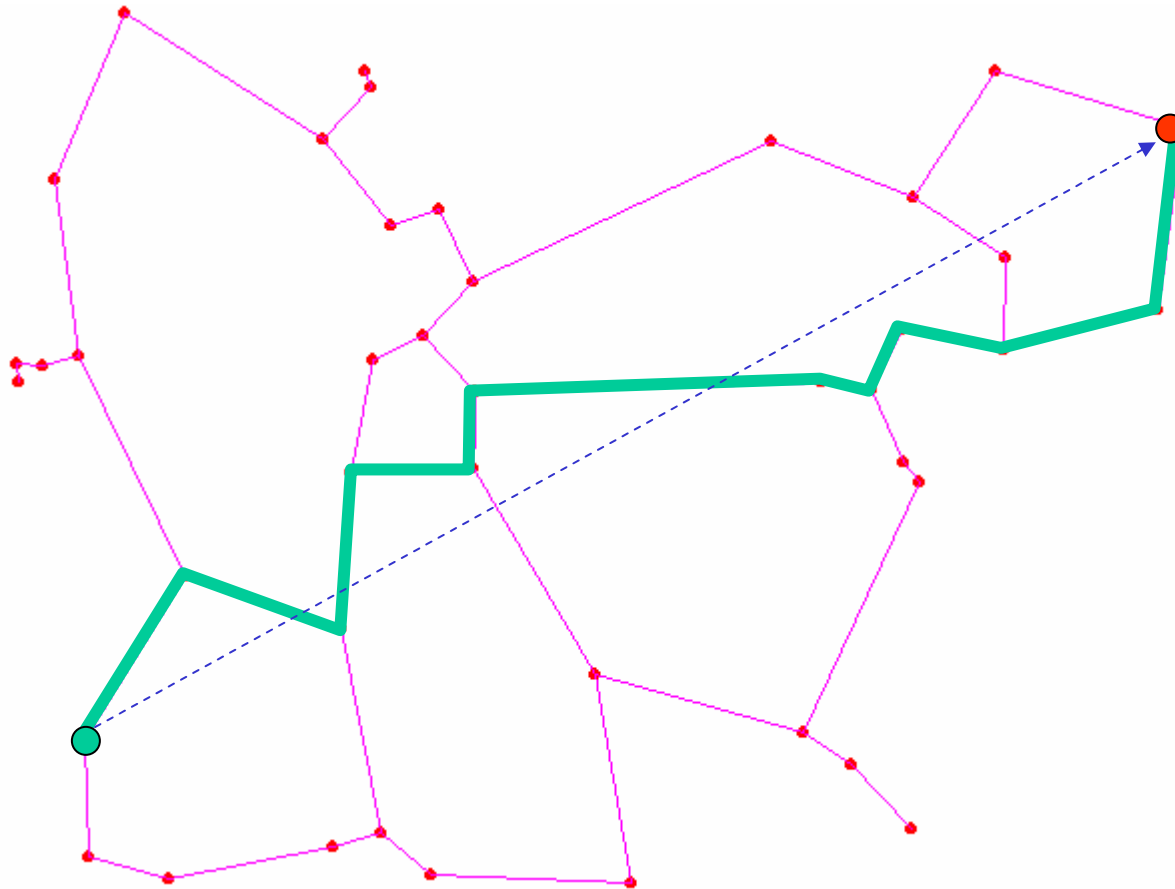
Greedy Routing



We greedily route to the neighbor which is closest to the target

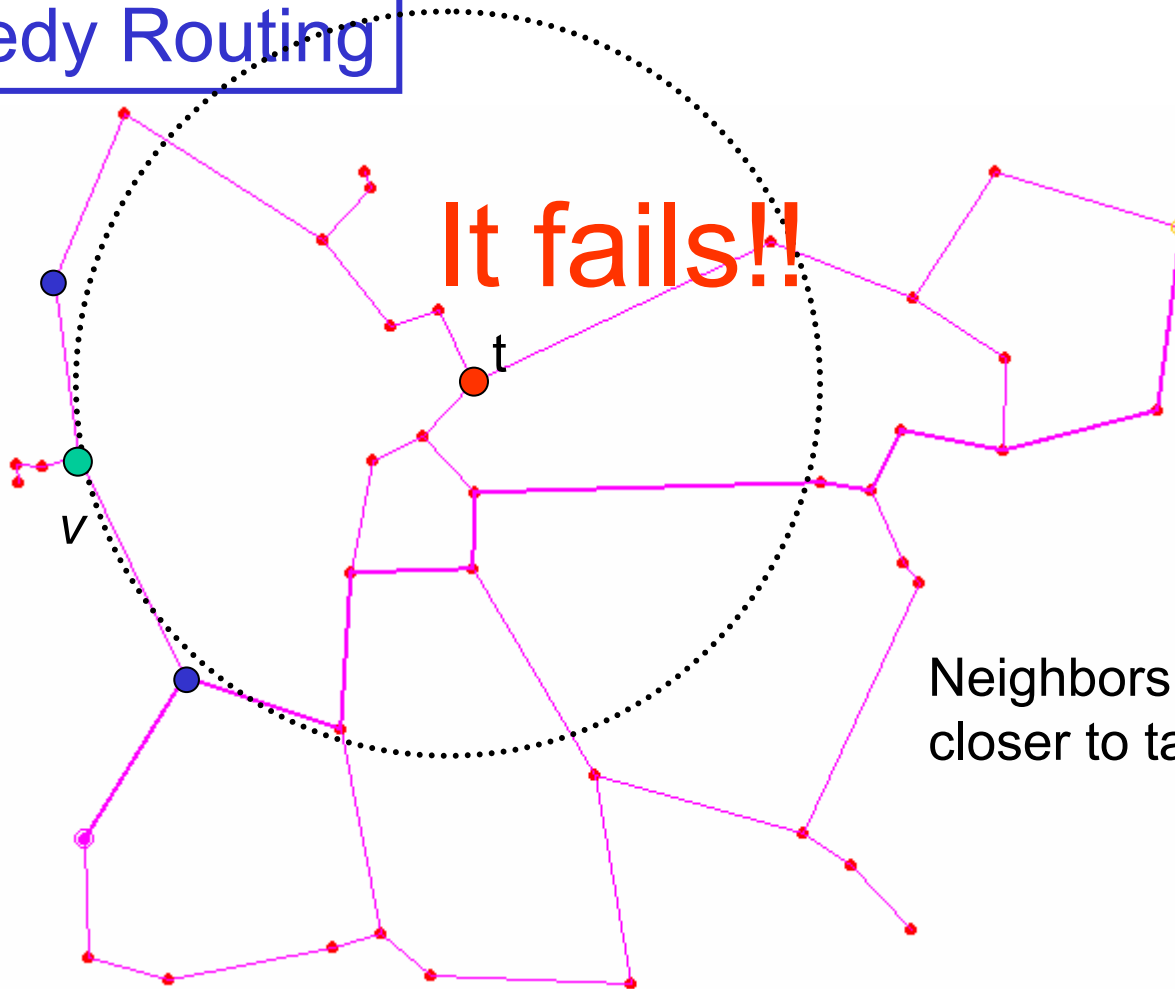
Wireless networks ROUTING

Greedy Routing



Wireless networks ROUTING

Greedy Routing



Neighbors of v are not closer to target

Wireless networks ROUTING

Greedy Routing

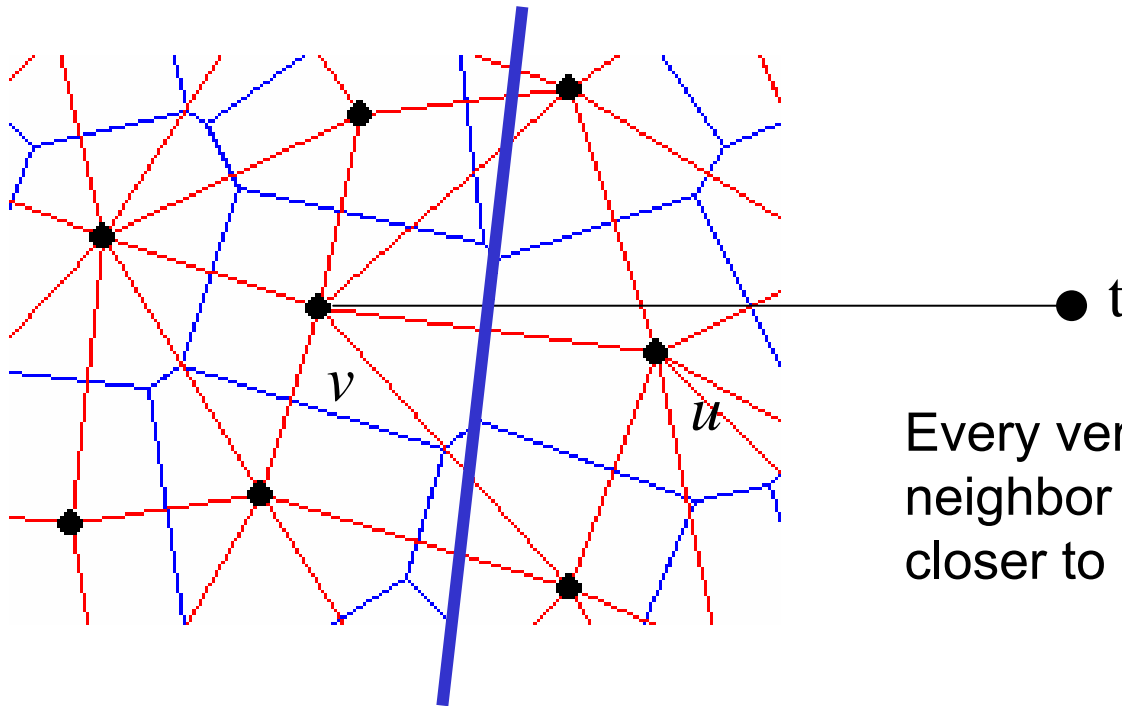
Memoryless algorithm

- Fails on some graphs
- Fails on some triangulations
- Always works for **D**elaunay **T**riangulations

Wireless networks ROUTING

Greedy Routing

Always works for **D**elaunay **T**riangulations

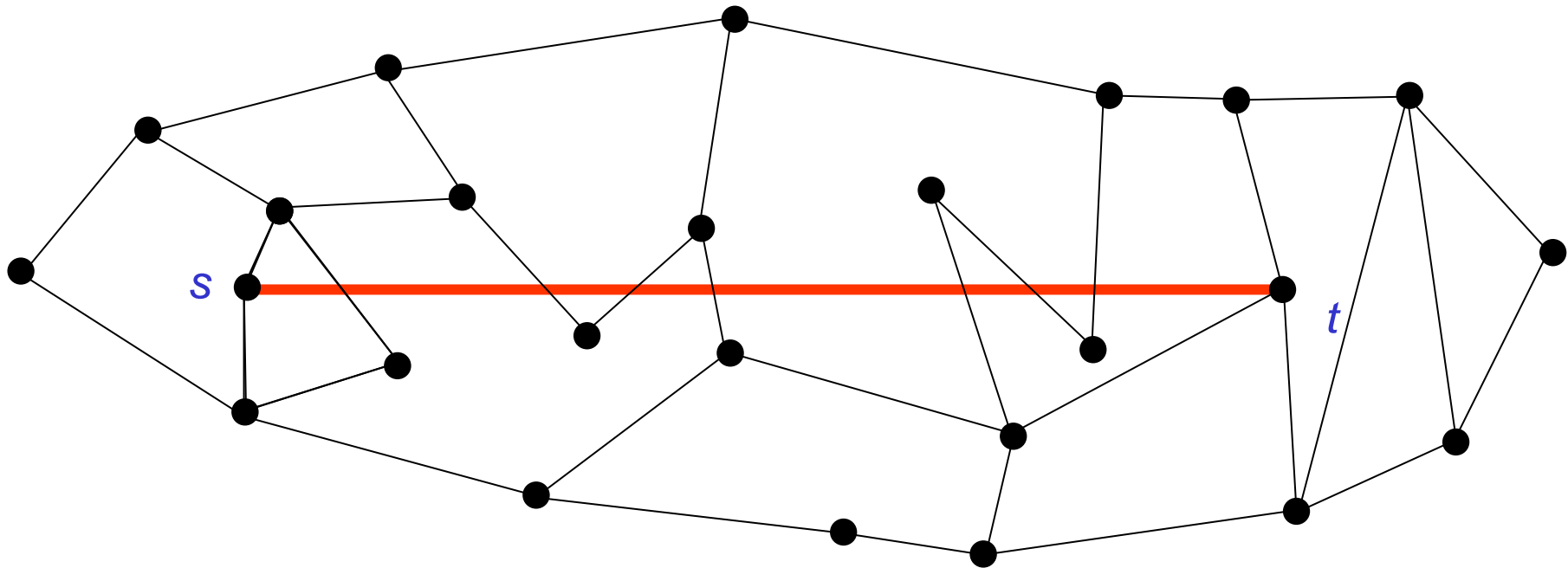


Every vertex v of **DT** has a neighbor that is strictly closer to t than v

Wireless networks ROUTING

Face Routing

Kranakis, Singh, Urrutia, '99

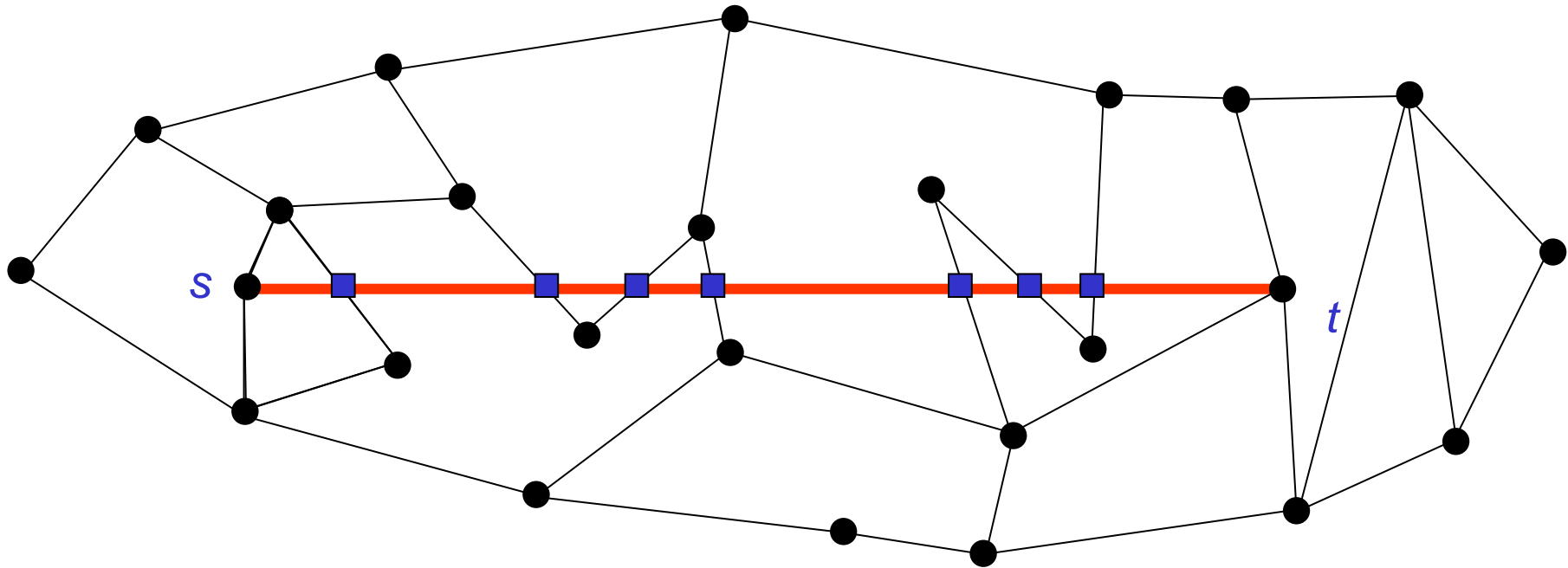


Route along the boundaries of the faces that lie on the source-target line st

Wireless networks ROUTING

FACE ROUTING

1. Let F be the face incident to the source s , intersected by line s,t



2. Explore the boundary of F ; remember the point p where the boundary intersects with (s,t) which is nearest to t .
Go back to p , switch the face and repeat step 2 until you hit the target t

Wireless networks ROUTING

FACE ROUTING

Theorem

Face routing terminates on any simple planar graph in $O(n)$ steps, where n is the number of the nodes in the network

Proof:

It is straightforward to deduce that we reach the destination t

We can order the faces that intersect the (s,t) line, therefore we never visit a face twice.

Each edge is in at most two faces, therefore each edge is visited at most 4 times.

Since a simple planar graph has at most $3n-6$ edges, the algorithm terminates in $O(n)$ steps.

Wireless networks ROUTING

FACE ROUTING

Theorem

Face routing terminates on any simple planar graph in $O(n)$ steps, where n is the number of the nodes in the network

Constant memory algorithm

Vertex of the face closest to target
Last vertex reached

Wireless networks ROUTING

ROUTING PROBLEM

FACE ROUTING

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Wireless networks ROUTING

ROUTING PROBLEM

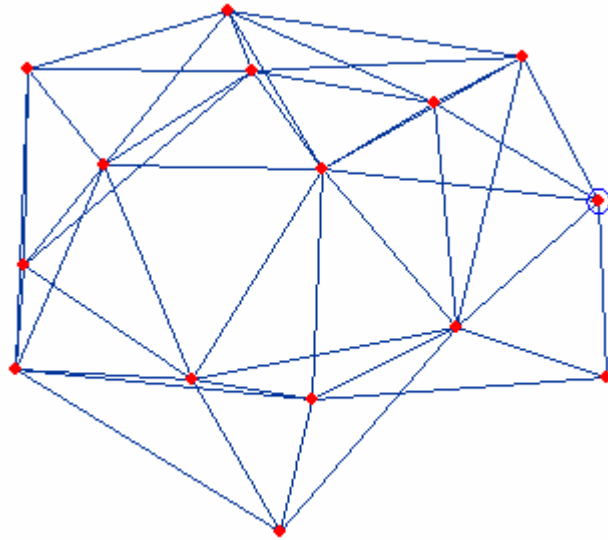
FACE ROUTING

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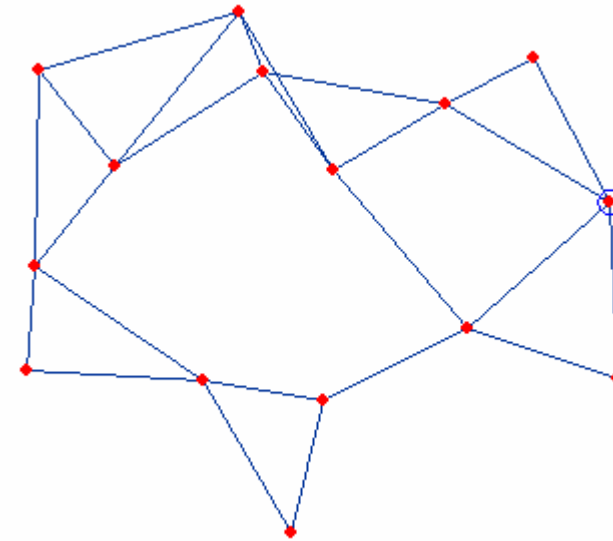
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Wireless networks



Unit Disk Graph

UDG



Unit Disk Graph, UDG, canNOT be a planar graph

Wireless networks

LOCAL ALGORITHM FOR ROUTING

PROBLEM 2

Given a UDG network N , find a local algorithm to extract a planar subgraph H such that if N is connected, then the subgraph H is also connected.

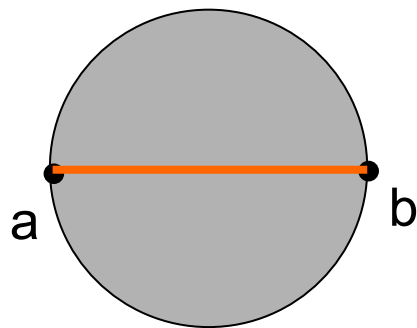
Solving problem 2 and using face routing in the resulting planar subgraph would immediately give an on-line local algorithm for routing in UDG networks

Gabriel Graph

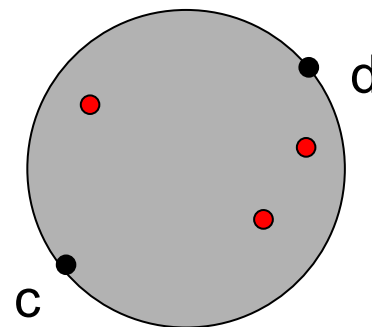
Proximity graphs

GG

p, q are adjacents if the circle of diameter pq does not contain in its interior other points of S



a, b are Gabriel neighbors



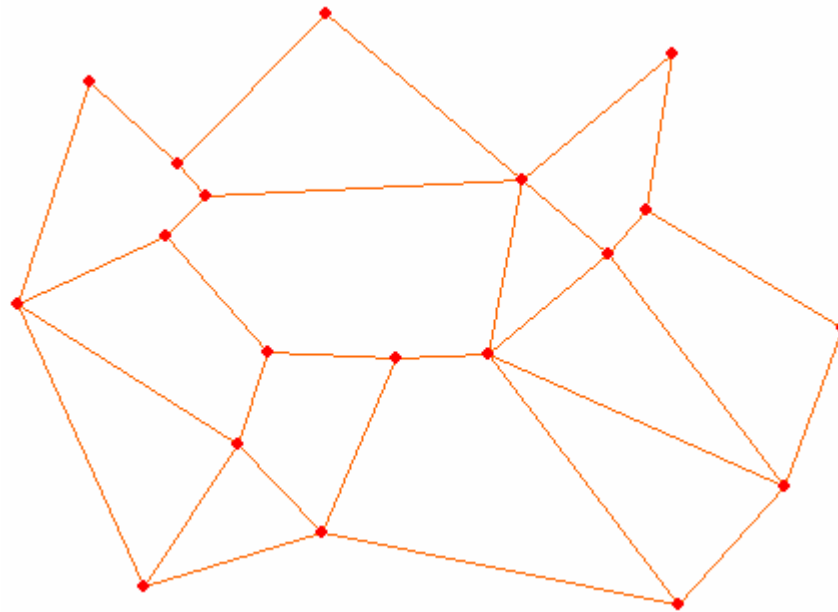
c, d are not Gabriel neighbors

Gabriel Graph

Gabriel Graph

GG

p, q are adjacents if the disk of diameter pq does not contain in its interior other points of S



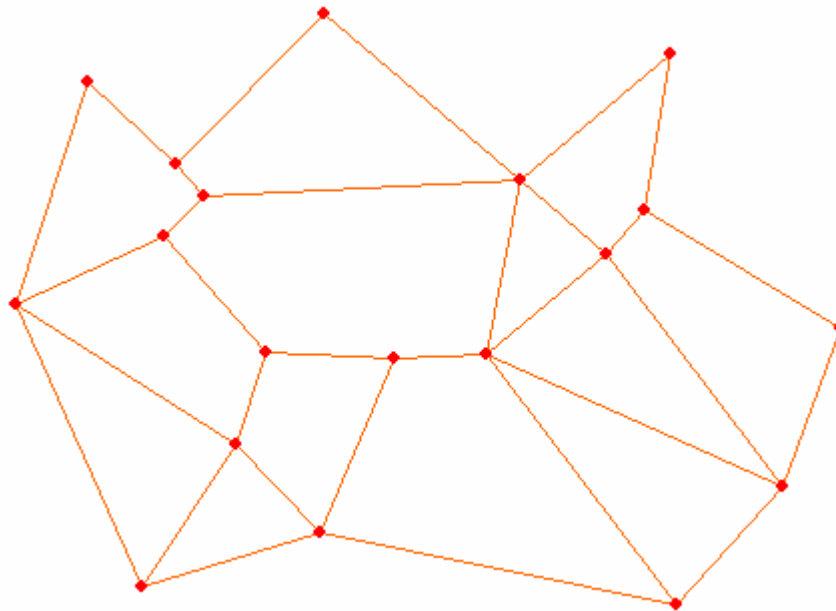
Gabriel Graph

Gabriel Graph

GG

p, q are adjacent if the disk of diameter pq does not contain in its interior other points of S

$DT \supset GG \supset EMST$



Unit Disk Graph UDG

Problem 2

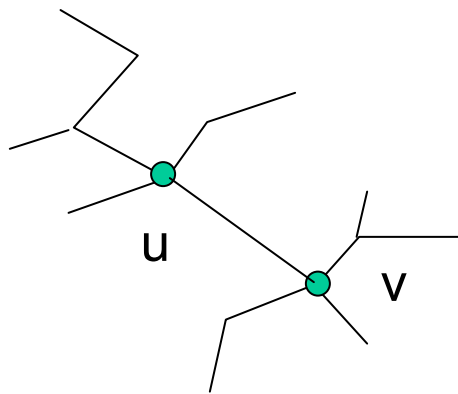
Given $UDG(S)$, construct a planar subgraph locally

Lemma 1

Given S , if $UDG(S)$ is connected, then $UDG(S) \cap GG$ is connected

Proof: Let $T = MST(S)$, we know that GG contains T . Therefore it is sufficient to prove that if UDG is connected then it contains $T = MST(S)$

We suppose there exists an edge uv in T with length > 1



$T - (uv)$ has two components, T_u, T_v

Unit Disk Graph UDG

Problem 2

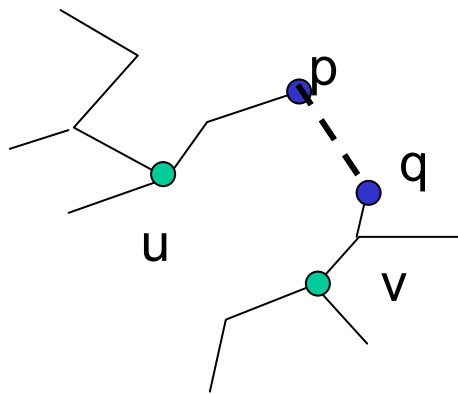
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We suppose there exists an edge uv in T with $length > 1$



$T - (uv)$ has two components, T_u, T_v

UDG connected, then exists pq , $|pq| < 1$

The tree $T - (uv) + (pq)$ weighs less than T !!

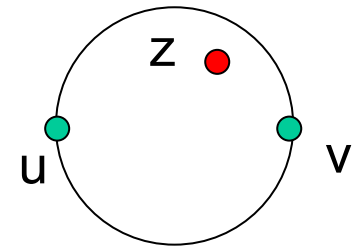
Unit Disk Graph UDG

Problem 2

Given $UDG(S)$, construct a planar subgraph locally

If (uv) is an edge of $UDG(S)$ and $(uv) \notin GG(S)$
then exists a point z inside the disk of diameter uv

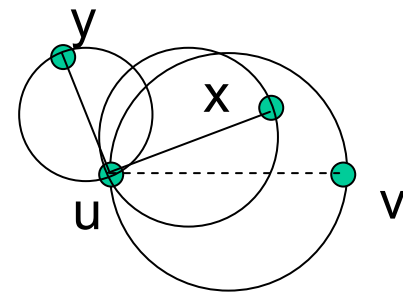
witness



Lemma 2

If $(uv) \notin GG(S)$ and z is a witness, then the edges uz and vz
belong to $UDG(S)$

Each vertex u deletes in $UDG(S)$ the edges
which do not belong to $GG(N(u) \cup \{u\})$



Unit Disk Graph UDG

Problem 2

Given UDG(S), construct a planar subgraph locally

Algorithm GABRIEL-UDG

For each neighbor $x \in N(u)$

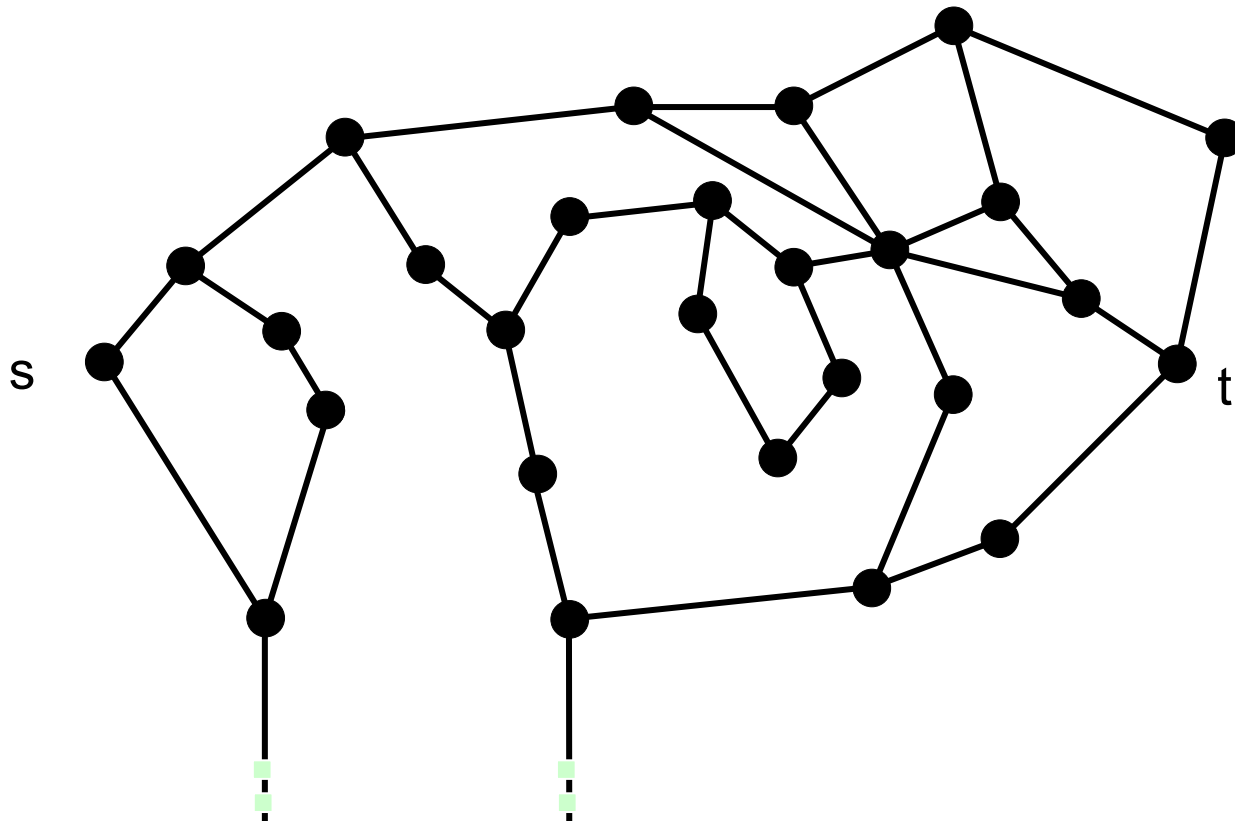
if $\text{disc}(u,x) \cap (N(u) - \{u,v\}) \neq \emptyset$ then delete the edge ux

The cost in each vertex u is $O(d^2)$ where $d = \text{degree}(u)$

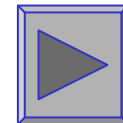
Wireless networks ROUTING

FACE ROUTING and GABRIEL GRAPH solve the problem, but
... the path, is a good path?

Can be very bad compared to the optimal route

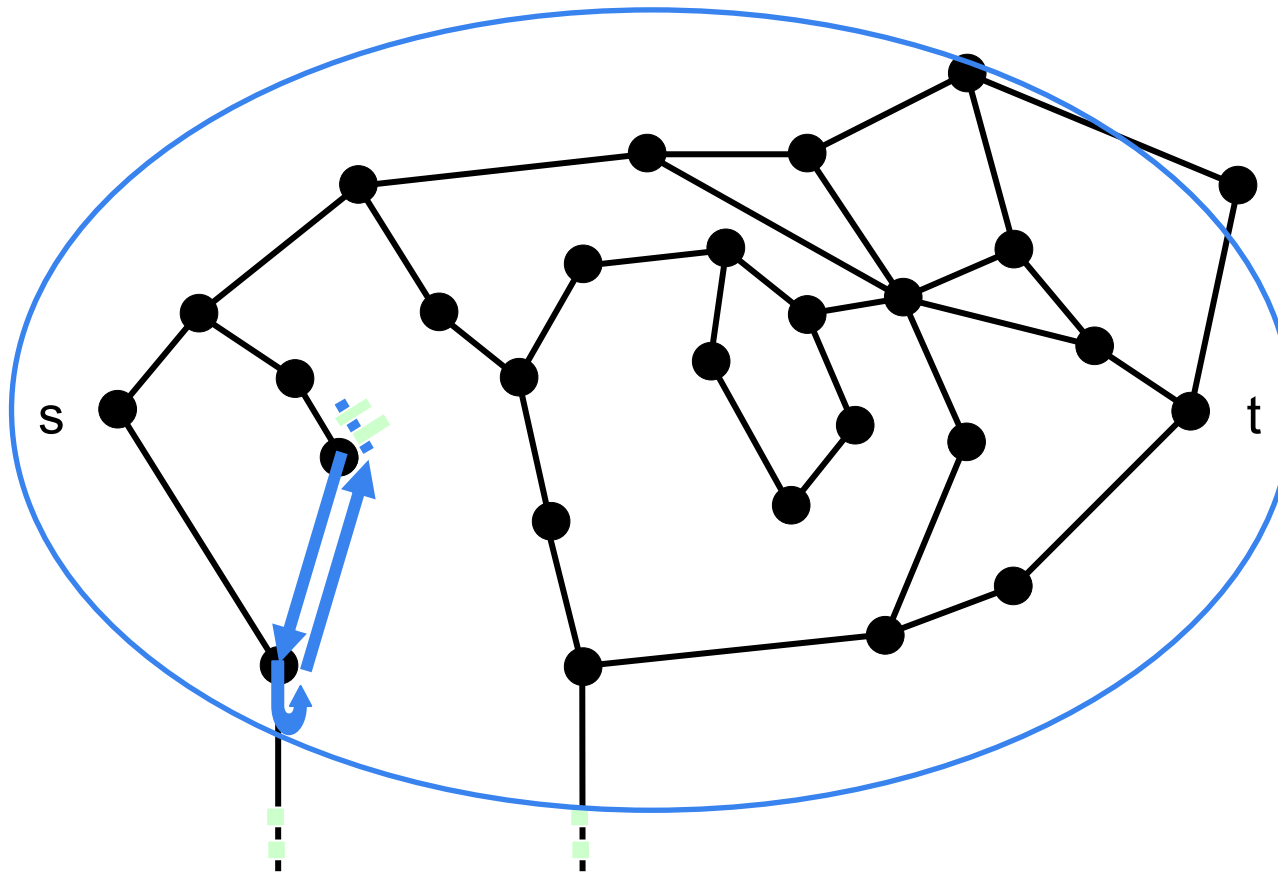


How to improve
face routing?



Wireless networks ROUTING

ADAPTIVE FACE ROUTING AFR



Wireless networks ROUTING

ADAPTIVE FACE ROUTING AFR

Analysis

Theorem

If the optimal s-t route in the UDG has cost k , then AFR terminates with a route whose cost is $O(k^2)$.

“cost” $c = c$ hops

Wireless networks ROUTING

LOWER BOUND

Let k be the length of the optimal path between s and t . There are networks for which the route found by any local algorithm has cost $\Omega(k^2)$

Kuhn, Wattenhofer, '02

Wireless networks ROUTING

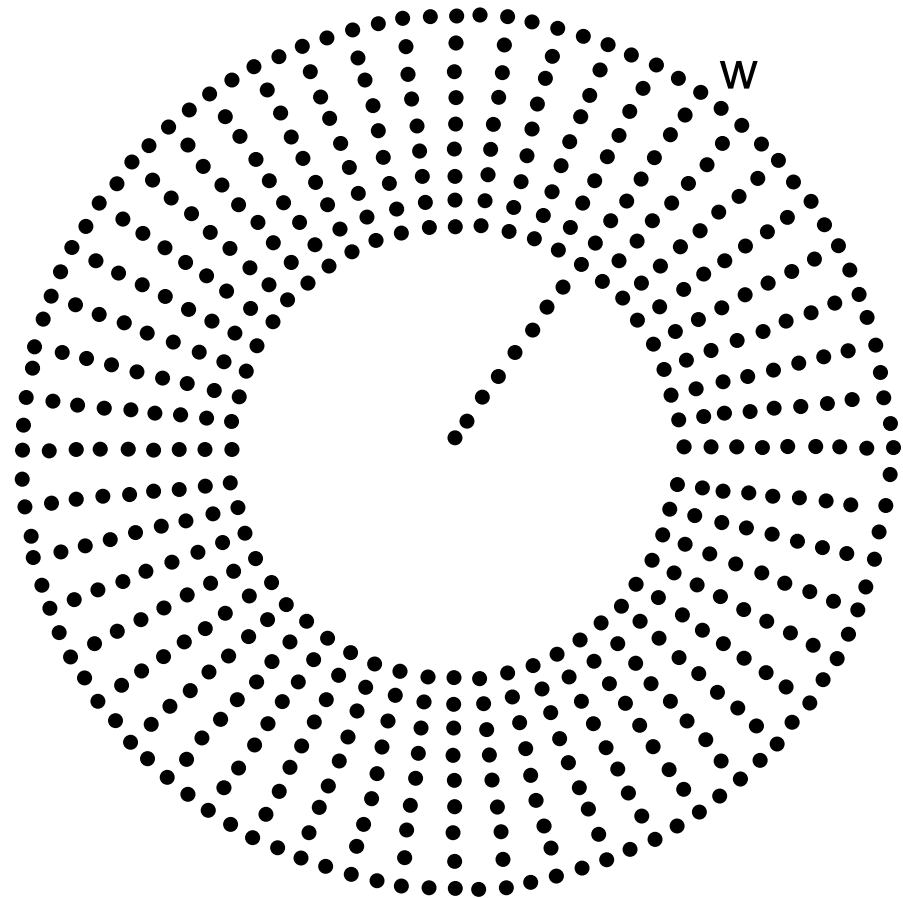
LOWER BOUND

On a circle we evenly distribute $2k$ nodes such that the distance between two neighboring points is exactly 1.

For every second node of the circle we construct a chain of $k/2\pi - 1$ nodes arranged on a line pointing towards the center.

The distance between two neighbors of a chain is exactly 1

Node w is one taken arbitrarily on the circle. The chain of w consists of k/π nodes with distance 1



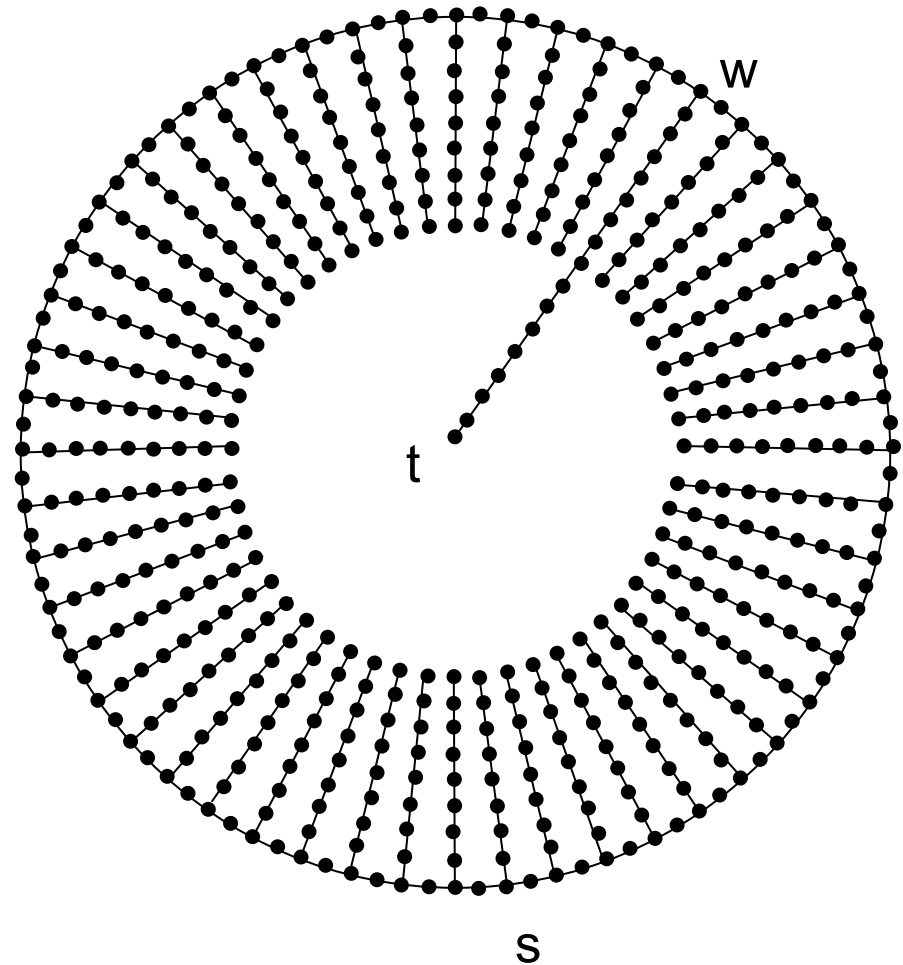
Wireless networks ROUTING

LOWER BOUND

The unit disk graph has k chains with $\Theta(k)$ nodes each.

We route from an arbitrary node on the circle (the source s) to the center of the circle (the destination t).

An optimal route between s and t follows the shortest path on the circle to w , and then directly follows w 's chain to t with total length $c \in O(k)$



Wireless networks ROUTING

LOWER BOUND

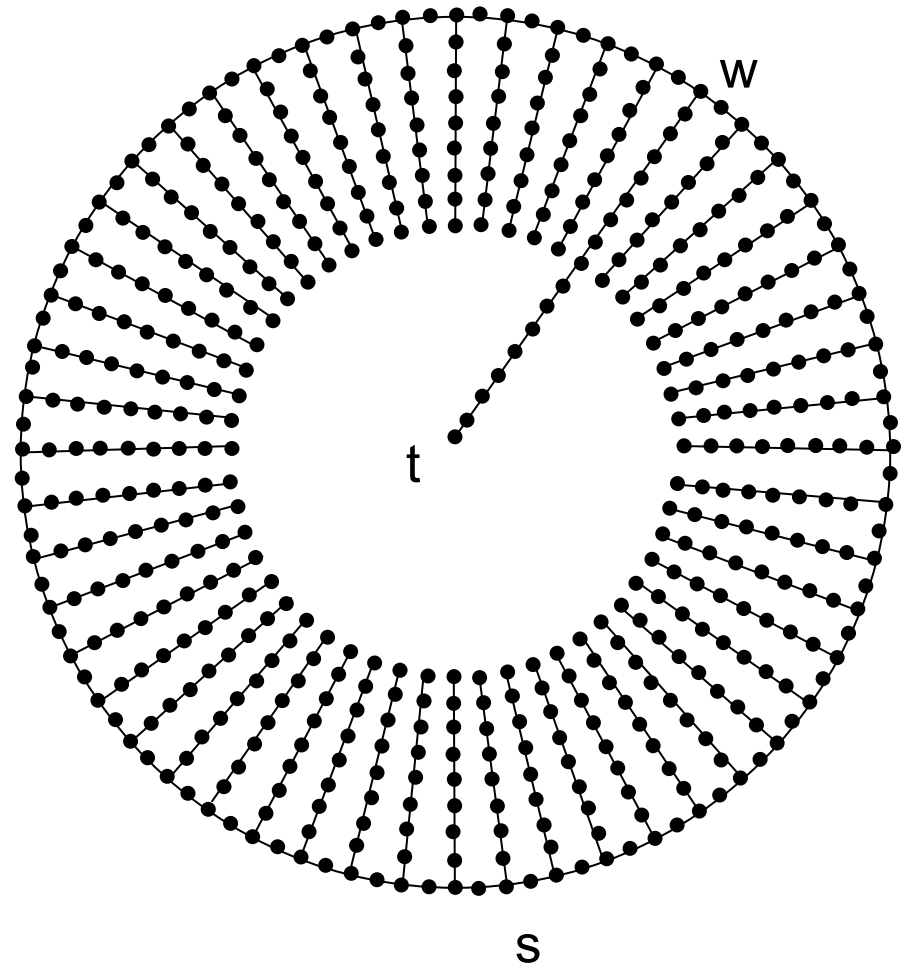
The unit disk graph has k chains with $\Theta(k)$ nodes each.

A geometric routing algorithm has to find the “correct” chain w .

Since there is no routing information stored at the nodes, this can only be achieved by exploring all the chains.

Any deterministic algorithm needs to explore all the chains until it finds the chain w .

The algorithm will therefore explore $\Theta(k^2)$ (instead of only $O(k)$) nodes



Wireless networks ROUTING

Theorem

Adaptive Face Routing is asymptotically optimal

Wireless networks ROUTING

ROUTING PROBLEM

Adaptive FACE ROUTING

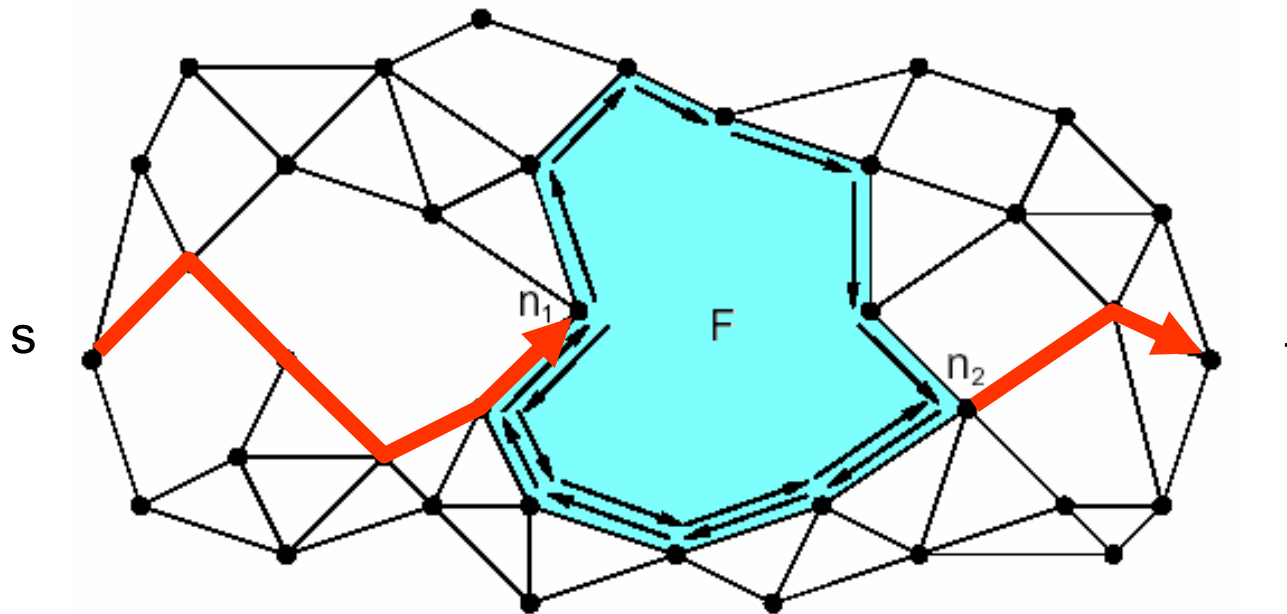
GREEDY ROUTING

GREEDY – Other Adaptive FACE ROUTING
GOAFR

Wireless networks ROUTING

ROUTING PROBLEM

**GREEDY – Other Adaptive FACE ROUTING
GOAFR**



1. Route greedily as long as possible
2. Circumvent “dead ends” by use of face routing
3. Then route greedily again

Wireless networks ROUTING

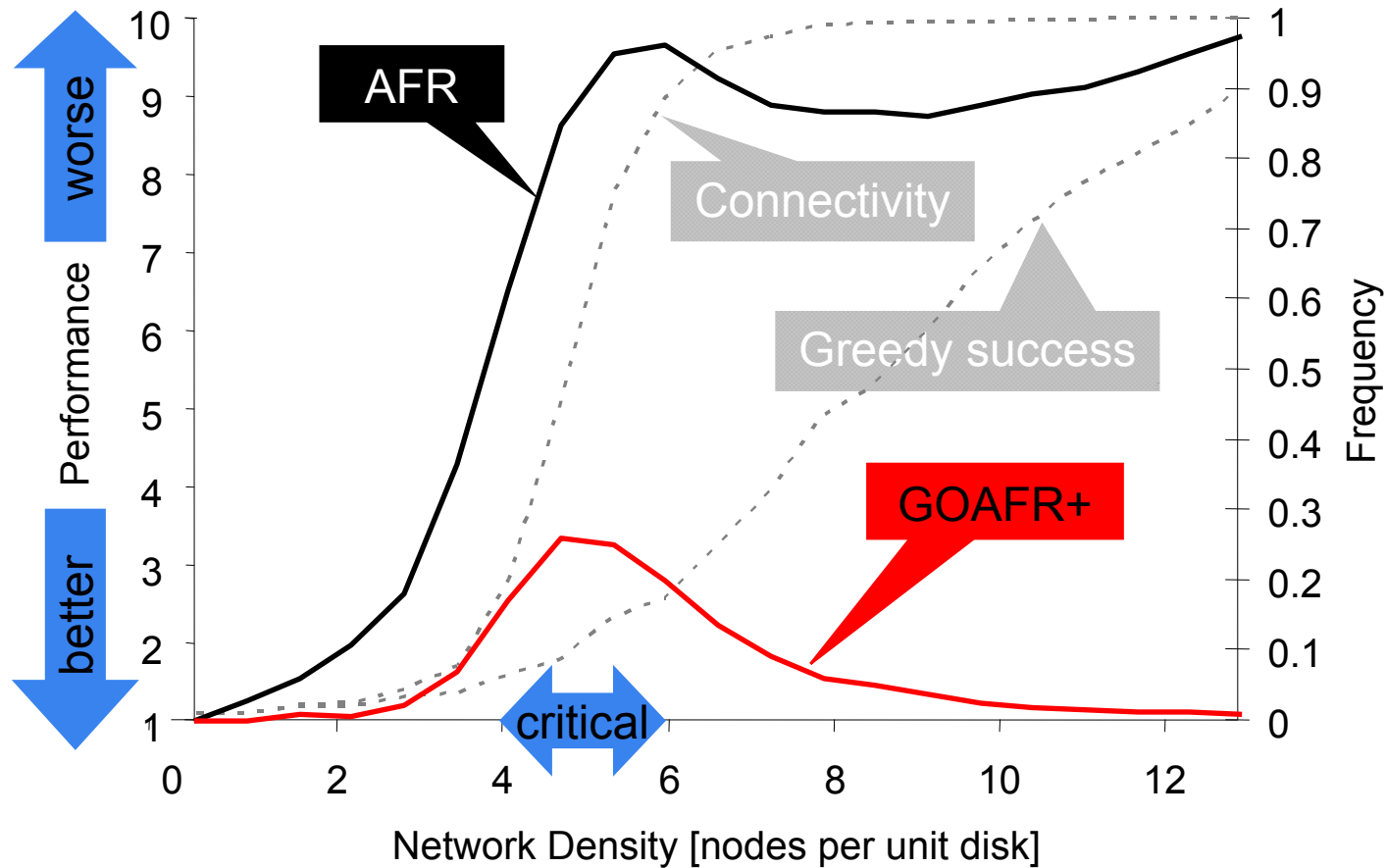
ROUTING PROBLEM

GREEDY – Other Adaptive FACE ROUTING
GOAFR

Theorem

GOAFR is still asymptotically worst-case optimal...
...*and* it is efficient in practice, in the average-case

Wireless networks ROUTING



Kuhn, Wattenhofer

Routing 3D

Wireless networks are in three-dimensional space

Can we translate 2D solutions to the 3D space?

- Greedy Routing?
- Face Routing?

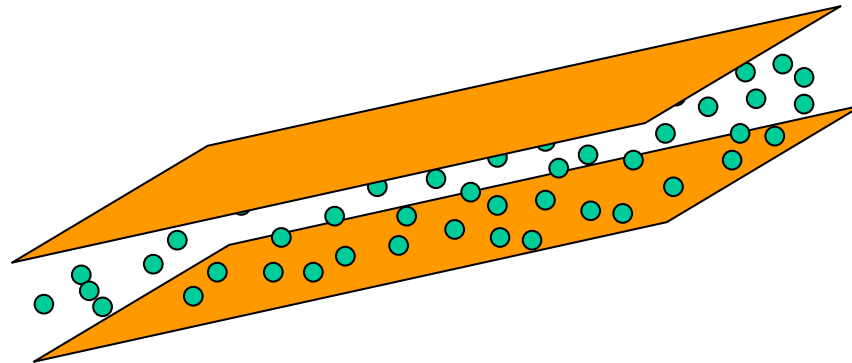
There is not local memoryless routing algorithm, that deliver messages deterministically in 3D

Durocher et al. 2008

Routing 3D

There is not local memoryless routing algorithm, that deliver messages deterministically in 3D

If the nodes are contained within a slab of thickness $1/\sqrt{2}$, there **exists** a 2-local routing algorithm that succeeds for UnitBallGraph(S).

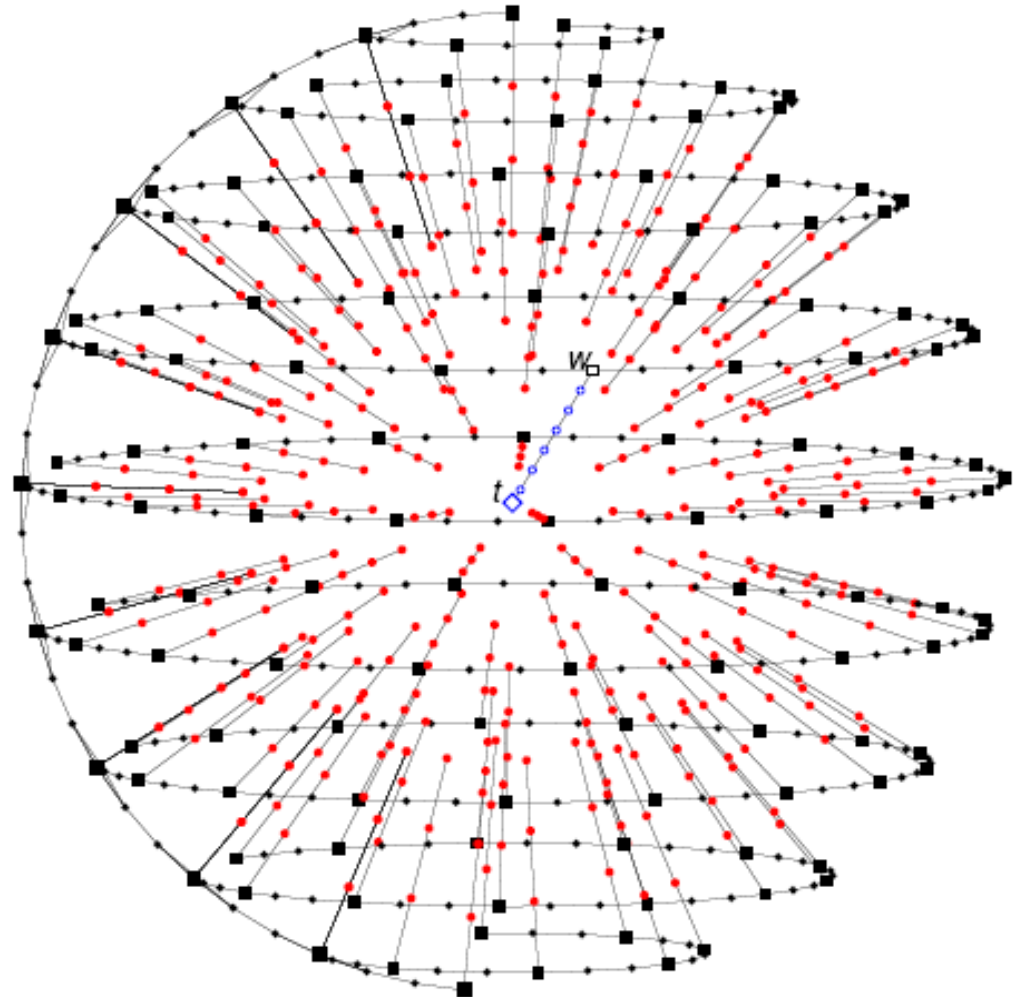


Routing 3D

and randomized?

There are networks for which the route found by any randomized geometric routing algorithm has expected length $\Omega(d^3)$

Flury, Wattenhofer, '08

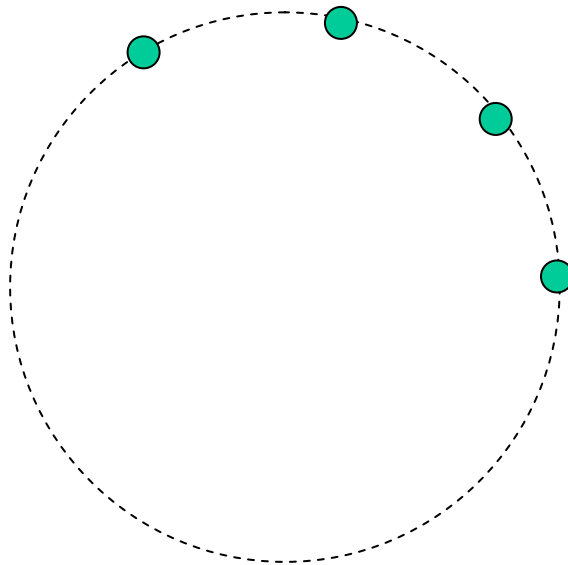


Local Algorithms

MST(UDG) Minimum Spanning Tree

It is NOT possible to calculate MST(S) with a local algorithm

S



n points on a circle C such that the distances between two consecutive nodes on C are $1 - \varepsilon_i$ $i=1, \dots, n$

Finding MST(S) is equivalent to identifying the smallest ε_i

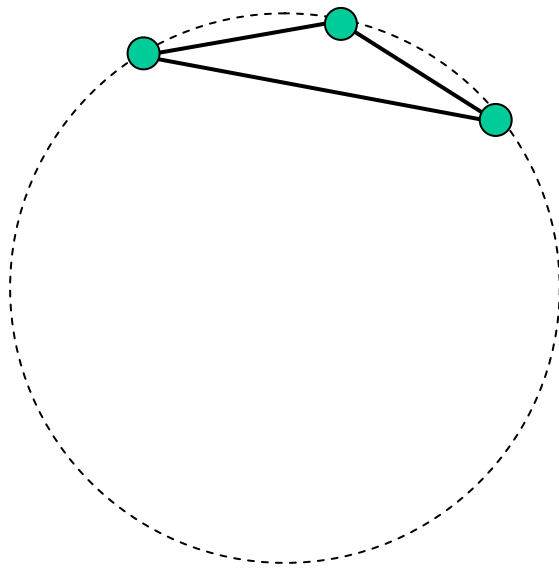
Therefore is not possible to obtain MST(S) locally

Local Algorithms

DT

It is NOT possible to calculate $DT(S)$ with a local algorithm

S



The circle may be arbitrarily large

Local Algorithms

UnitDiskGraph

Czyzowicz et al. '07, '08

- | | |
|----------------------------|------------|
| • Dominating set | 5-approx |
| • Connected Dominating Set | 7.4-approx |
| • Vertex Color | 7-approx |

Wiese, Kranakis, '08

PTAS for Independent Set, Vertex Cover, Dominating Set

Local Algorithms

Graphs

Kuhn, Moscibroda, Wattenhofer, '04

Many graph problems cannot be solved locally on general graphs.
Min Vertex Cover, Min Dominating Set, Max Independent Set, ...

Lenzen, Wattenhofer

'08 Dominating Set (planar graph), 74 -aprox.

'10 (bounded arboricity a) $O(a^2)$ -aprox.

Schneider, Wattenhofer

'10 MaxIndependentSet (bounded independence)

Polishchuk, Suomela, '09

Vertex Cover (bounded degree), 3 -aprox.

REFERENCES

- E. Kranakis, H. Singh, J. Urrutia, *Compass routing on geometric networks*, CCCG'99
- P. Morin, *Online routing in geometric graphs*, 2002
- F. Kuhn, R. Wattenhofer, A. Zollinger, DialM 2002, MobiHoc 2003
- N. Linial, *Locality in distributed graph algorithms*. Siam J. Comp. 1992
- J. Urrutia, *Local solutions for global problems in wireless networks*, J. Discrete Algorithms, 2007
- “Distributed Computing Group” ETH Zürich, <http://dcg.ethz.ch/index.html>