Universidad Politécnica
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Universidade de Aveiro

GEOMETRIC ROUTING

An introduction

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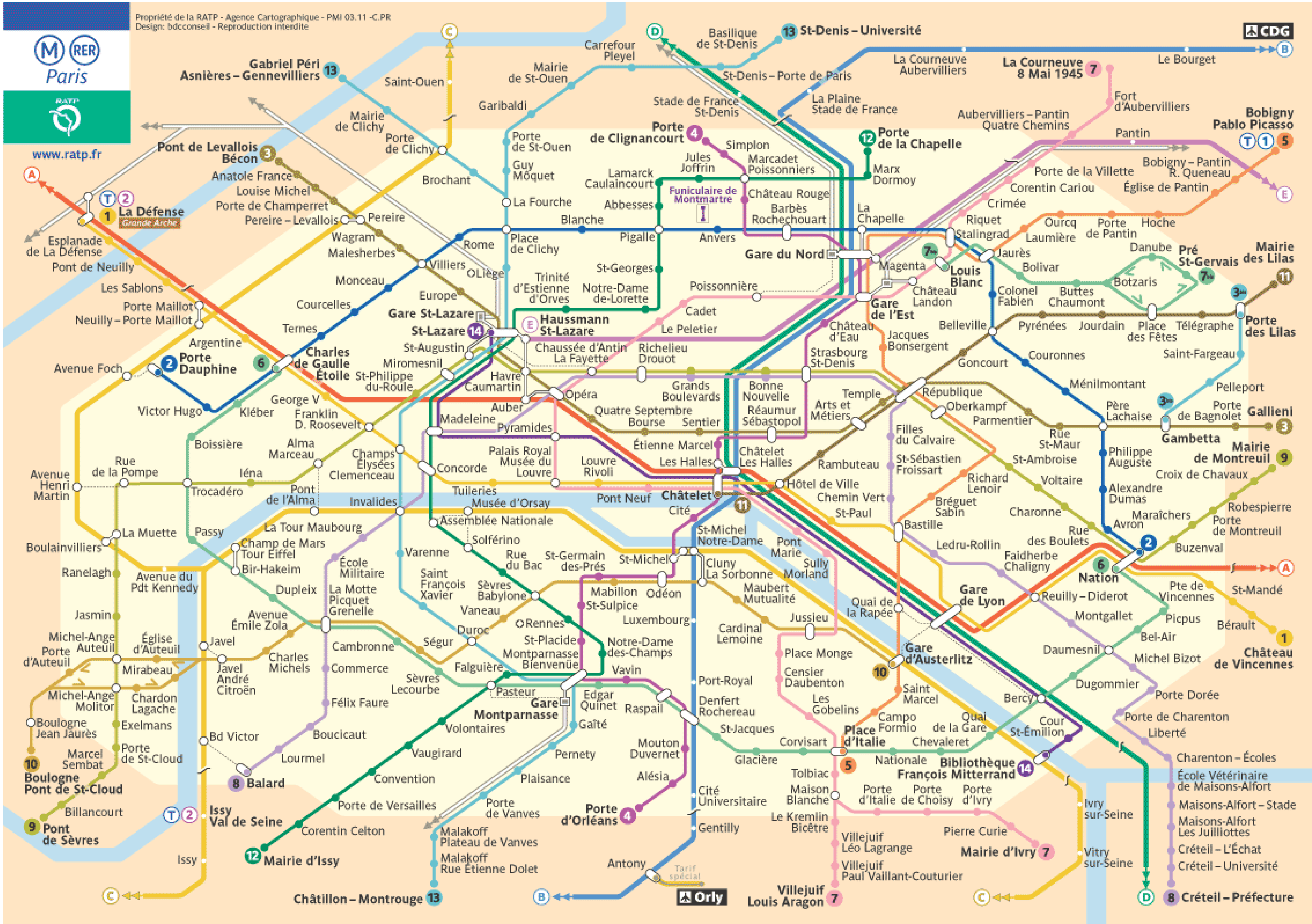
UPM

Aveiro 12-11-2004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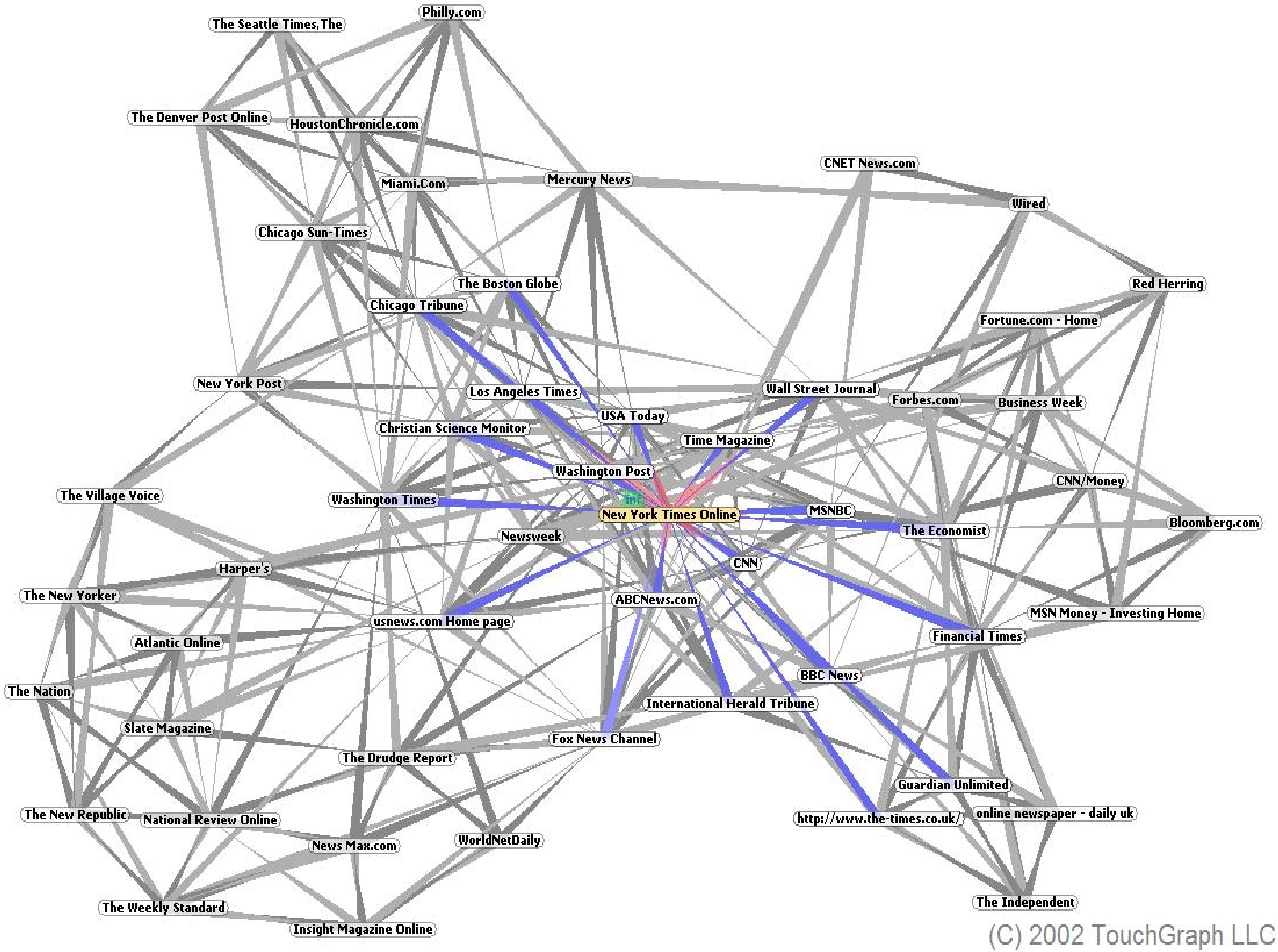
Vitry sur-Seine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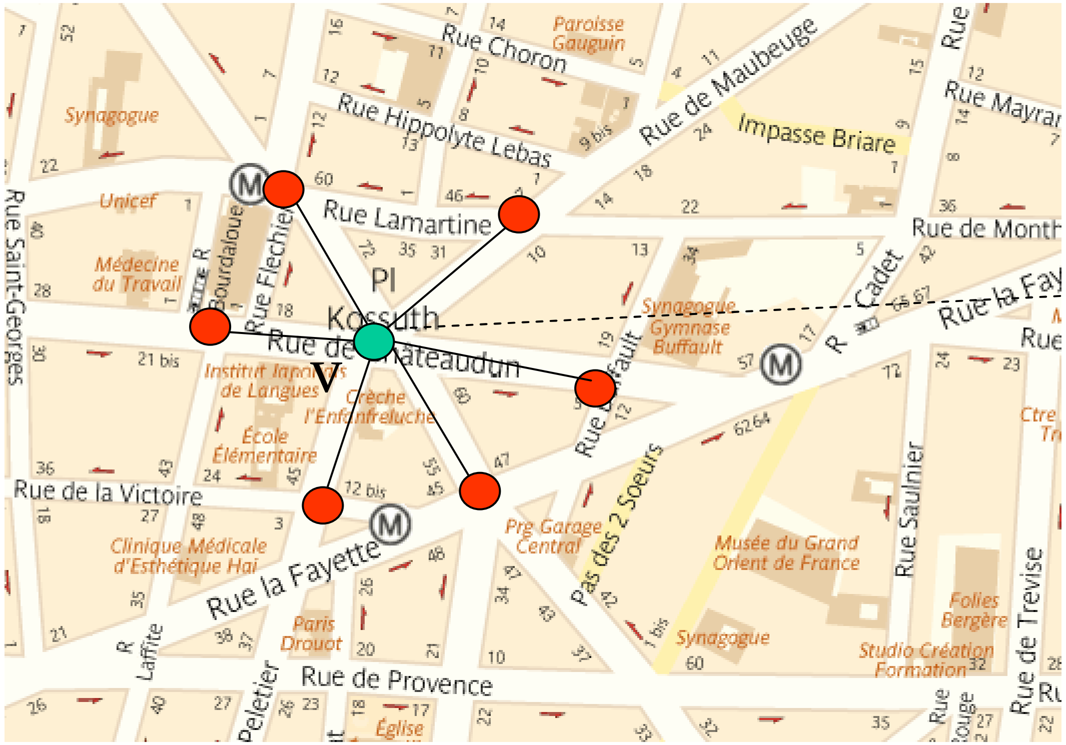
A tourist at Paris



Tour Eiffel

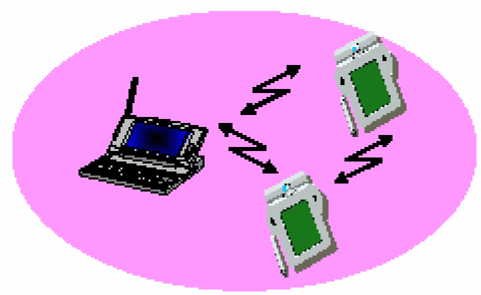
Local information (coordinates of v, target, and neighbors N(v))

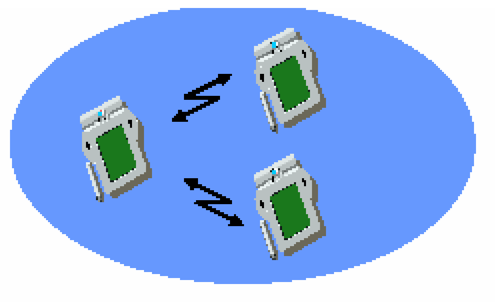
Limited memory allocation

Ecologically sound algorithms

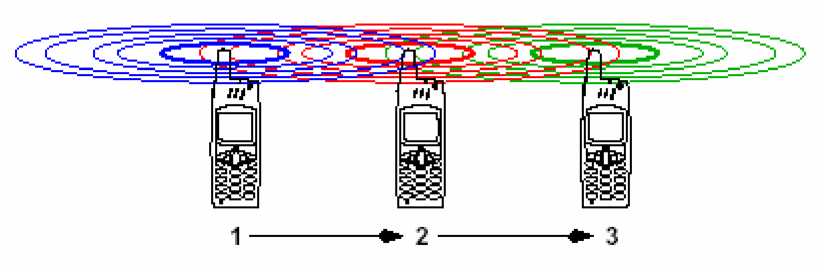
Mobile *ad hoc* Wireless Networks (MANET)

- Sometimes there is no infrastructure
  remote areas, unplanned meetings, disaster areas, ships in ocean



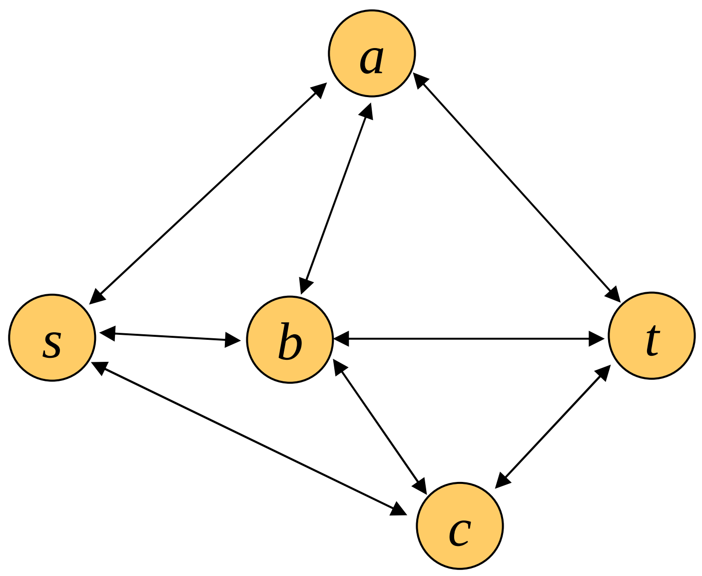


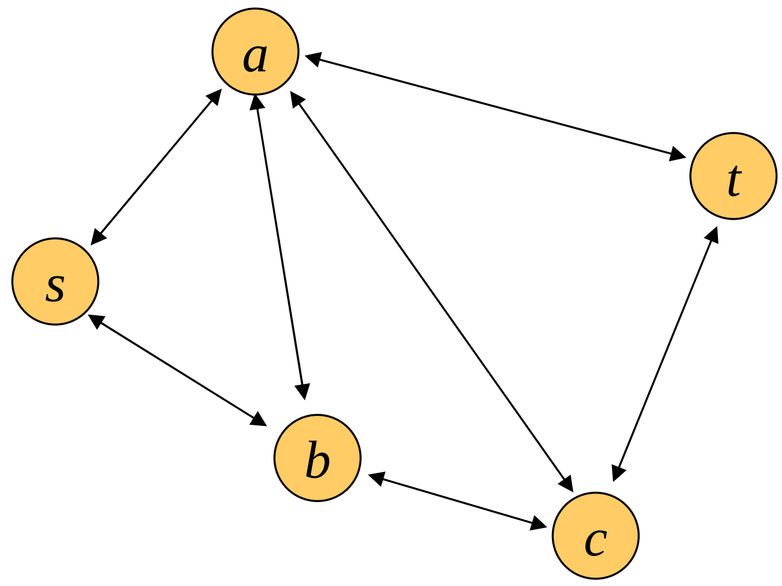
- Sometimes not every station can hear every other station
  Data needs to be forwarded in a multihop manner



Mobile *ad hoc* Wireless Networks (MANET)

Manet's consist of wireless hosts that communicate with each other in the absence of fixed infrastructure





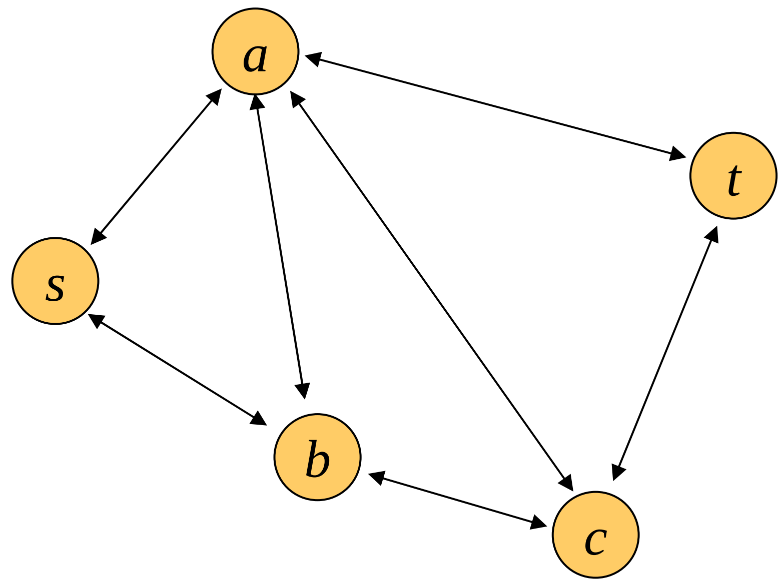
Nodes move!!

Mobile *ad hoc* Wireless Networks (MANET)

Manet's consist of wireless hosts that communicate with each other in the absence of fixed infrastructure

A MANET as a graph

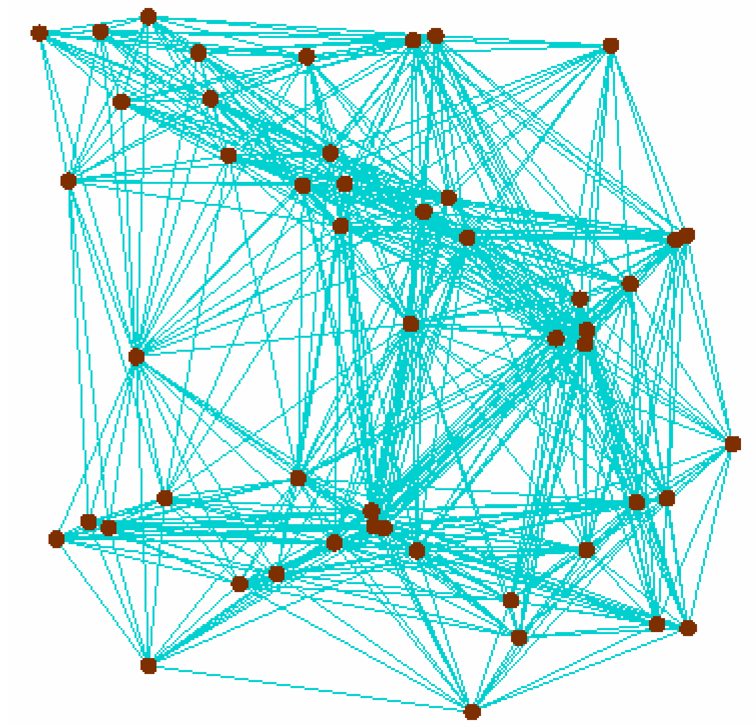
- A node is a mobile station
- If node *v* can "hear" node *u* there is an edge (*u*,*v*)
- The graph is euclidean (there is a link between two nodes iff the distance is less than their two broadcast ranges)



Mobile *ad hoc* Wireless Networks (MANET)

Unit disk graphs

Given a set of points V, UDG(V) is a geometric graph, in which there is an edge (u,v) iff $dist(u,v) \leq 1$



If all nodes have the same broadcast ranges, UDG are a model of MANETs

UDG has many edges

Overview

- Classic routing
- Geometric graphs
- Memoryless algorithms: greedy, compass, ..
- O(1)-memory algorithms: face, ..
- Competitive algorithms

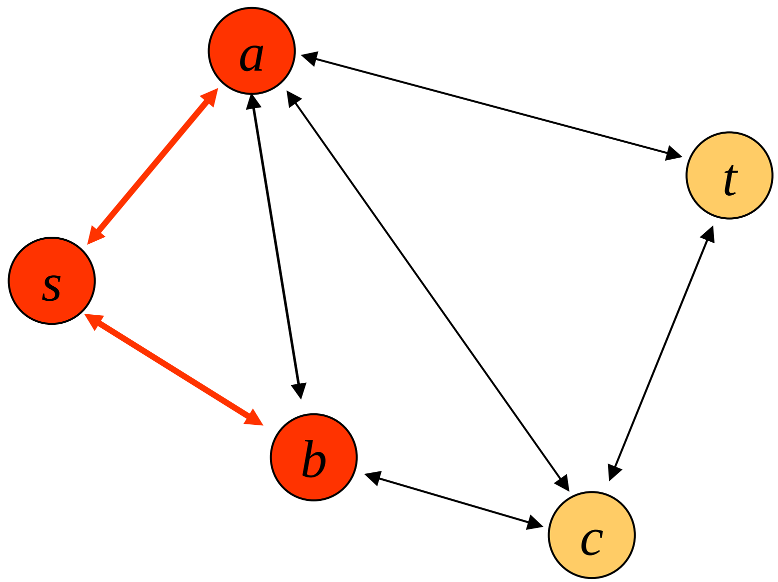
Classic Routing

Flooding

A source *s* sends the message to all its neighbors; when a node receive the message the first time it resends it to all its neighbors

Problems

- a node might see the same message more than once
- what if the network is huge and the target is next to *s*?



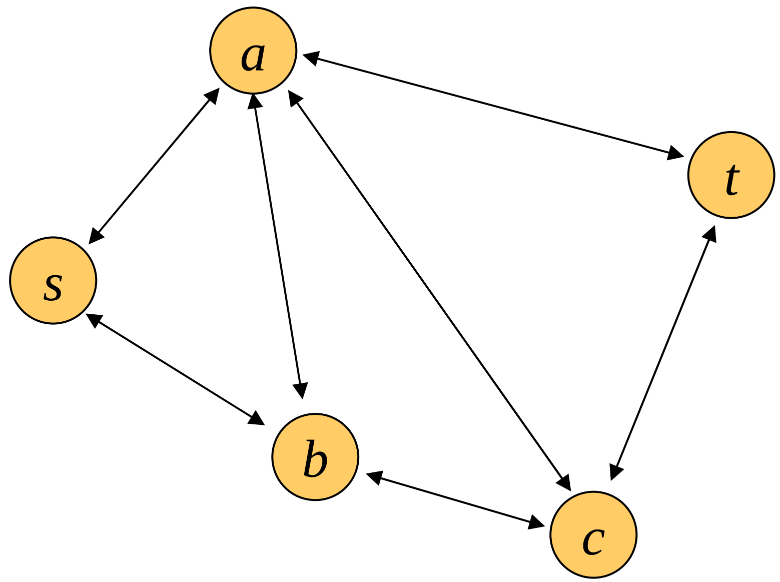
Classic Routing

Distance Vector

- Each node store a routing table that has an entry to each target
- If a node notices a change, it updates its routing table and sends an update to all neighbors, and their ....
- Message follows shortest path

Problems

- every node needs to store a big table

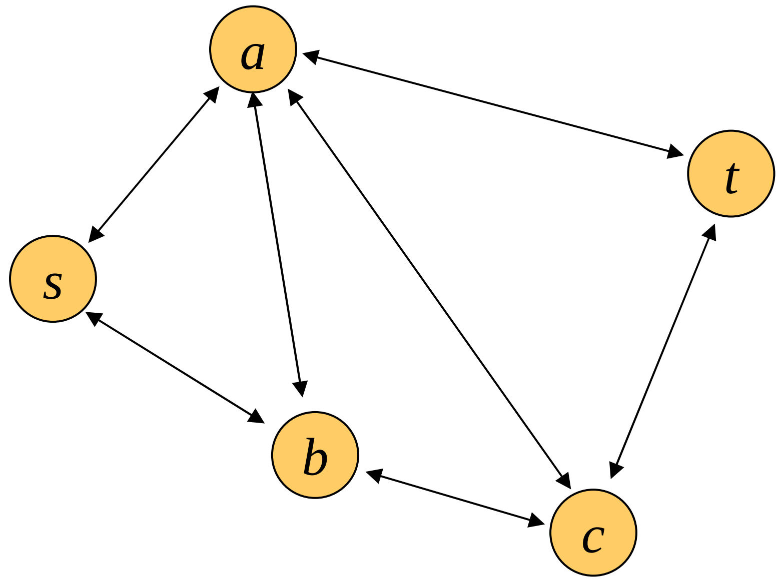


Geometric Routing

- Each node is equipped with a location service (they have GPS, Galileo, or an ad-hoc way to know their coordinates)
- Each node knows all the neighbor nodes and their coordinates
- The messenger knows the coordinates of the target

The messages travel
in a
geometric graph

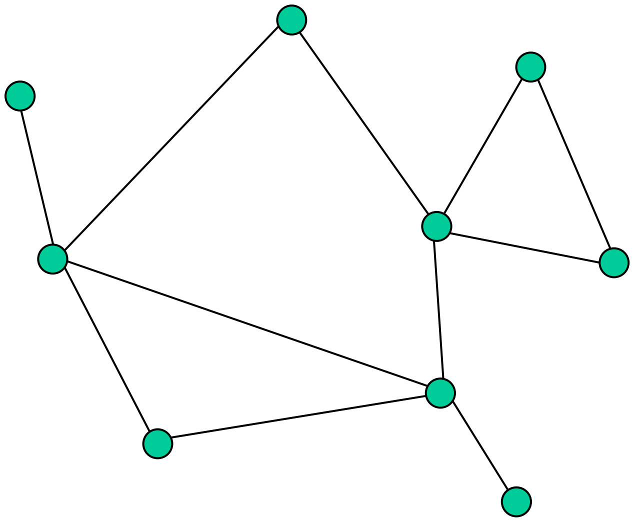
What is geometric graph?

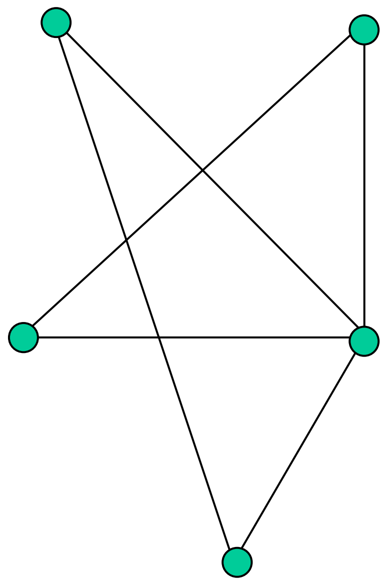


GEOMETRIC GRAPHS

Vertices $\rightarrow$ Points

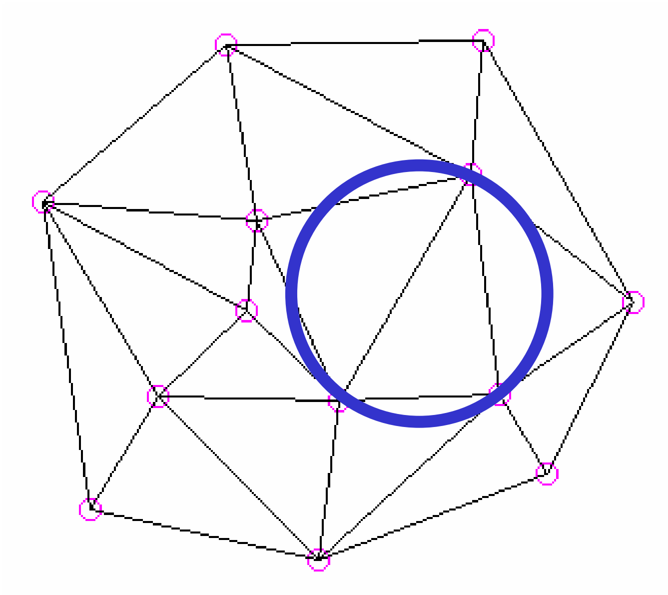
Edges $\rightarrow$ Segments





GEOMETRIC GRAPHS

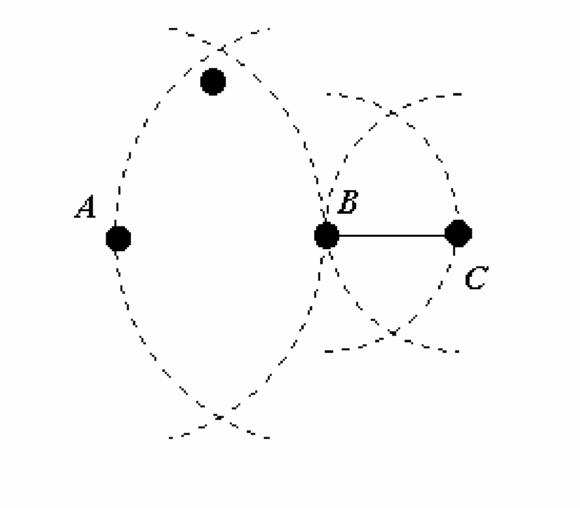
Triangulation of a set of points

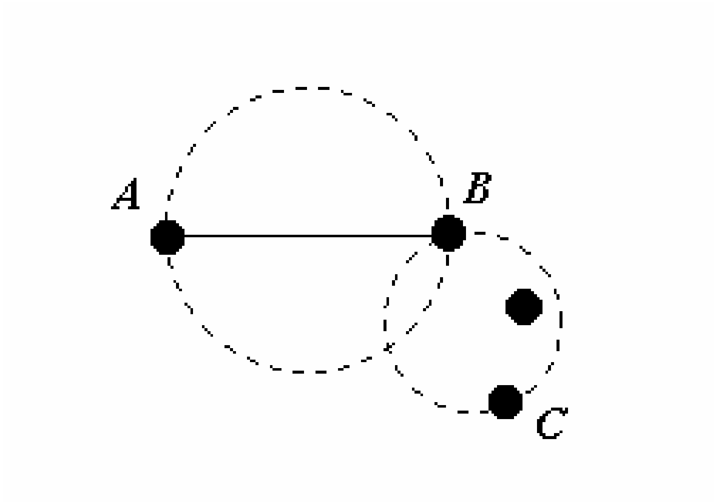


Delaunay triangulation

GEOMETRIC GRAPHS

Subgraphs of Delaunay Triangulation



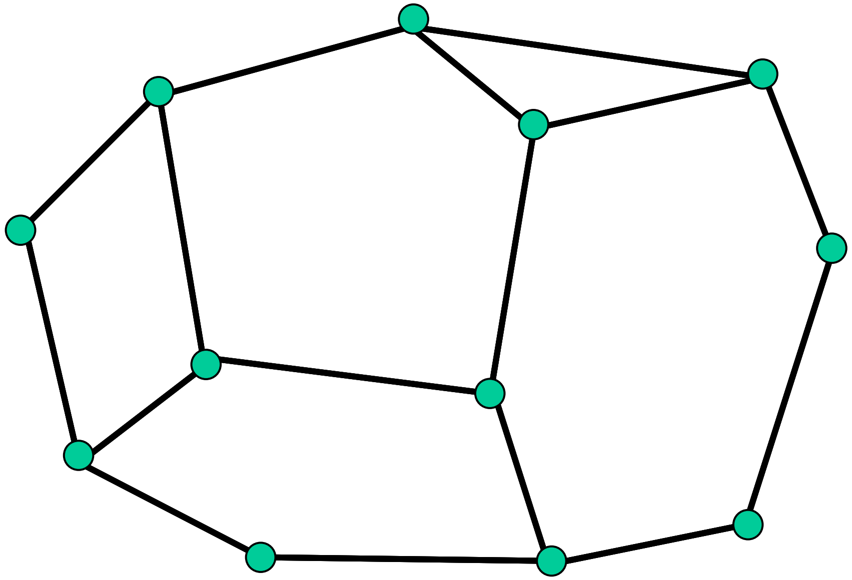


Relative Neighborhood Graph

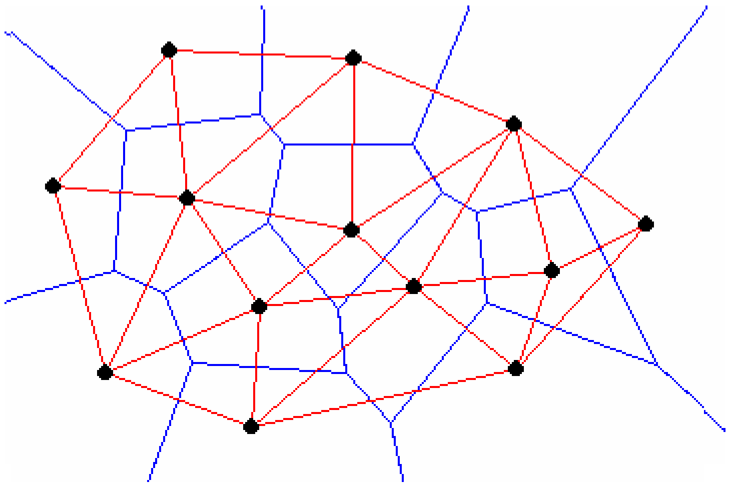
Gabriel Graph

GEOMETRIC GRAPHS

Convex subdivision



Voronoi Diagram



Dual of **DT**

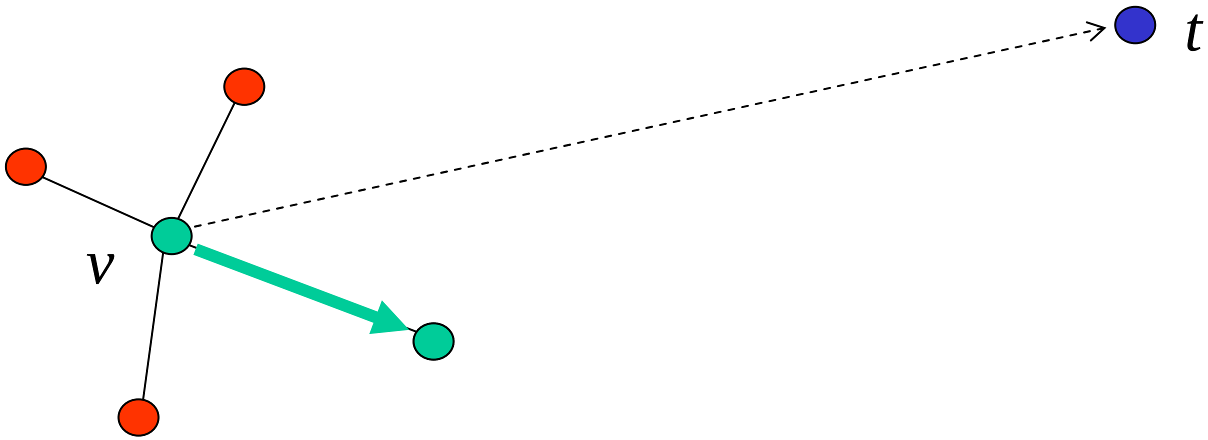
MEMORYLESS ALGORITHM

A routing algorithm is *memoryless* if the decision about which vertex to visit when we are situated at vertex *v* and destinated for *t* only depends of *v*, *t*, and *N(v)*

An algorithm **A** is defeated by a graph G if there exists a pair of vertices *s*,*t* such that a packet stored at *s* never reaches *t* when being routed using **A**.
Otherwise, we say that **A** works for G

Memoryless algorithm

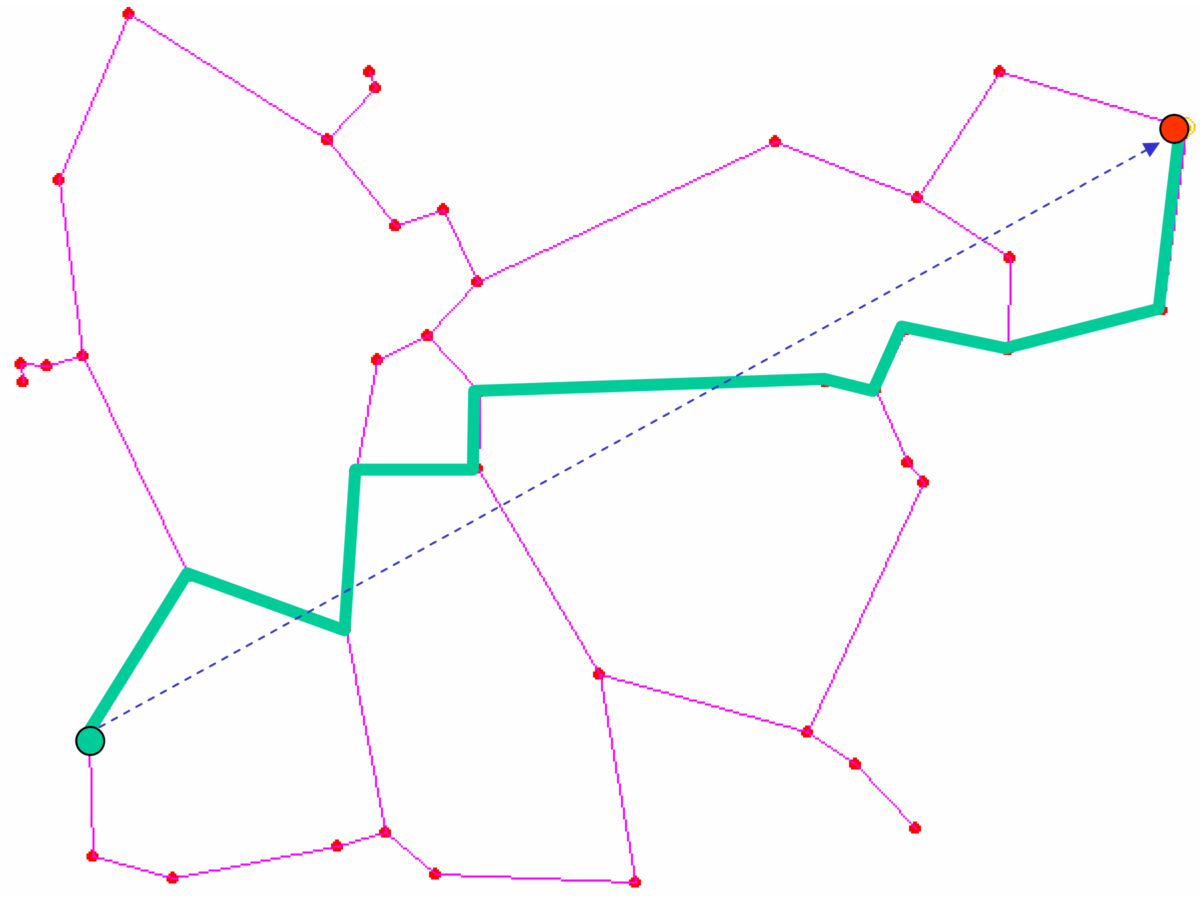
Greedy routing



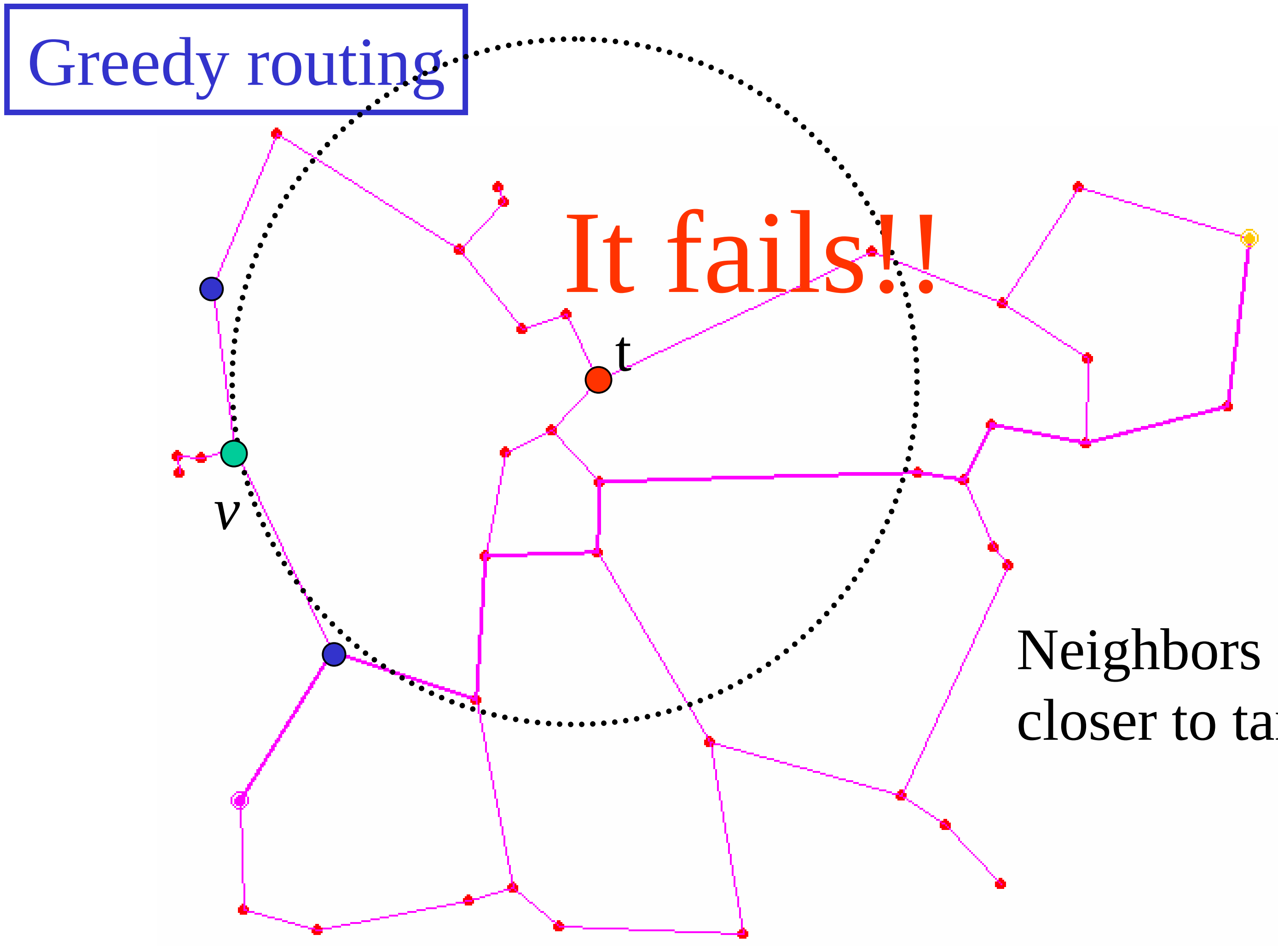
We greedily route to the neighbor which is closest to the target

Memoryless algorithm

Greedy routing



Memoryless algorithm

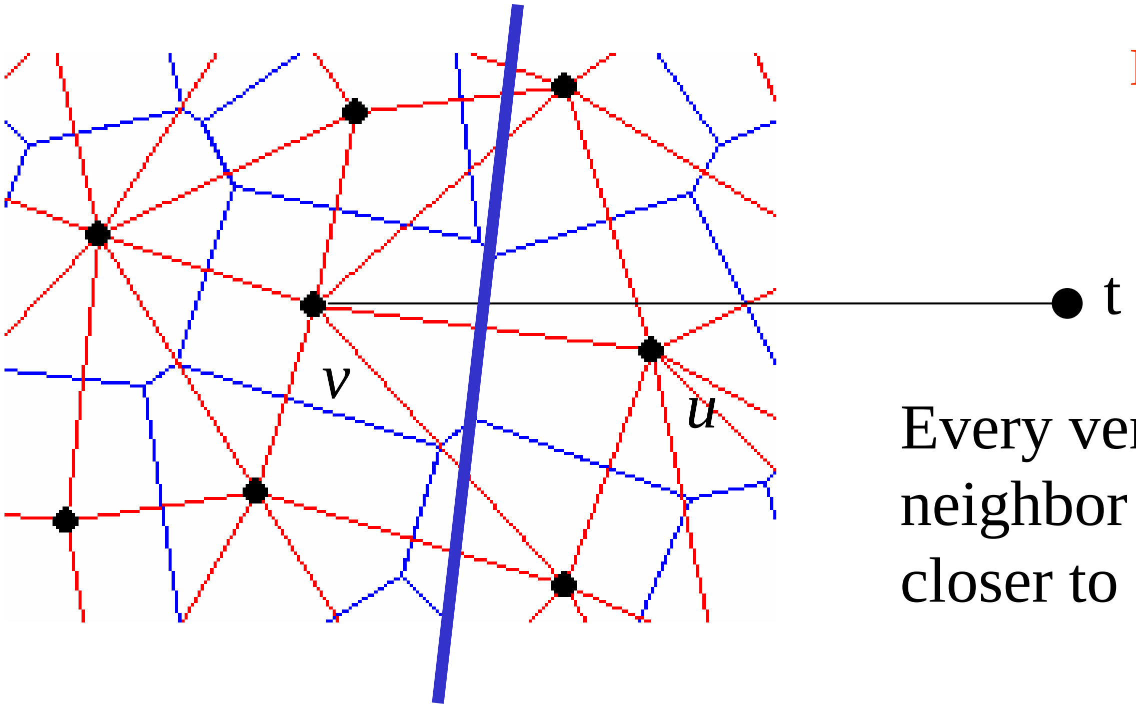


Neighbors of *v* are not closer to target

Memoryless algorithm

Greedy routing

- Fails on some graphs
- Fails on some triangulations
- Always works for Delaunay Triangulations



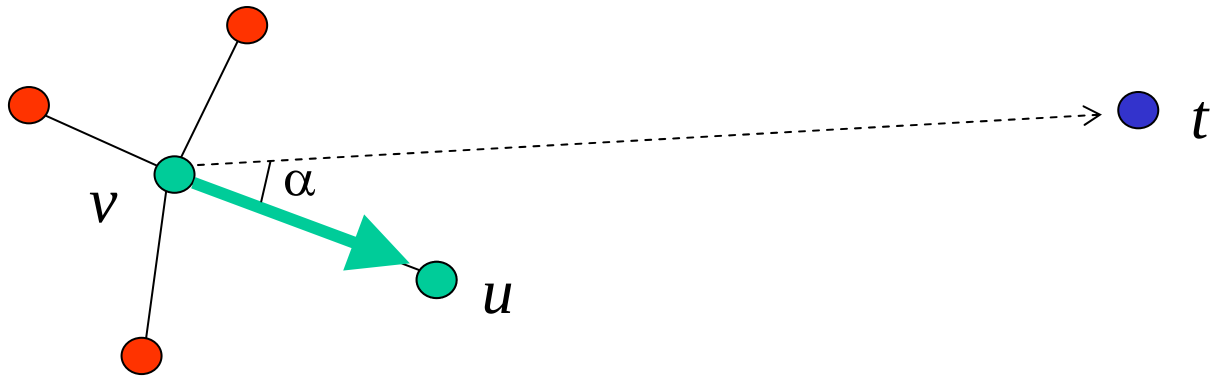
Bose, Morin 99

Every vertex *v* of **DT** has a neighbor that is strictly closer to *t* than *v*

Memoryless algorithm

Compass Routing

Kranakis, Urrutia 99

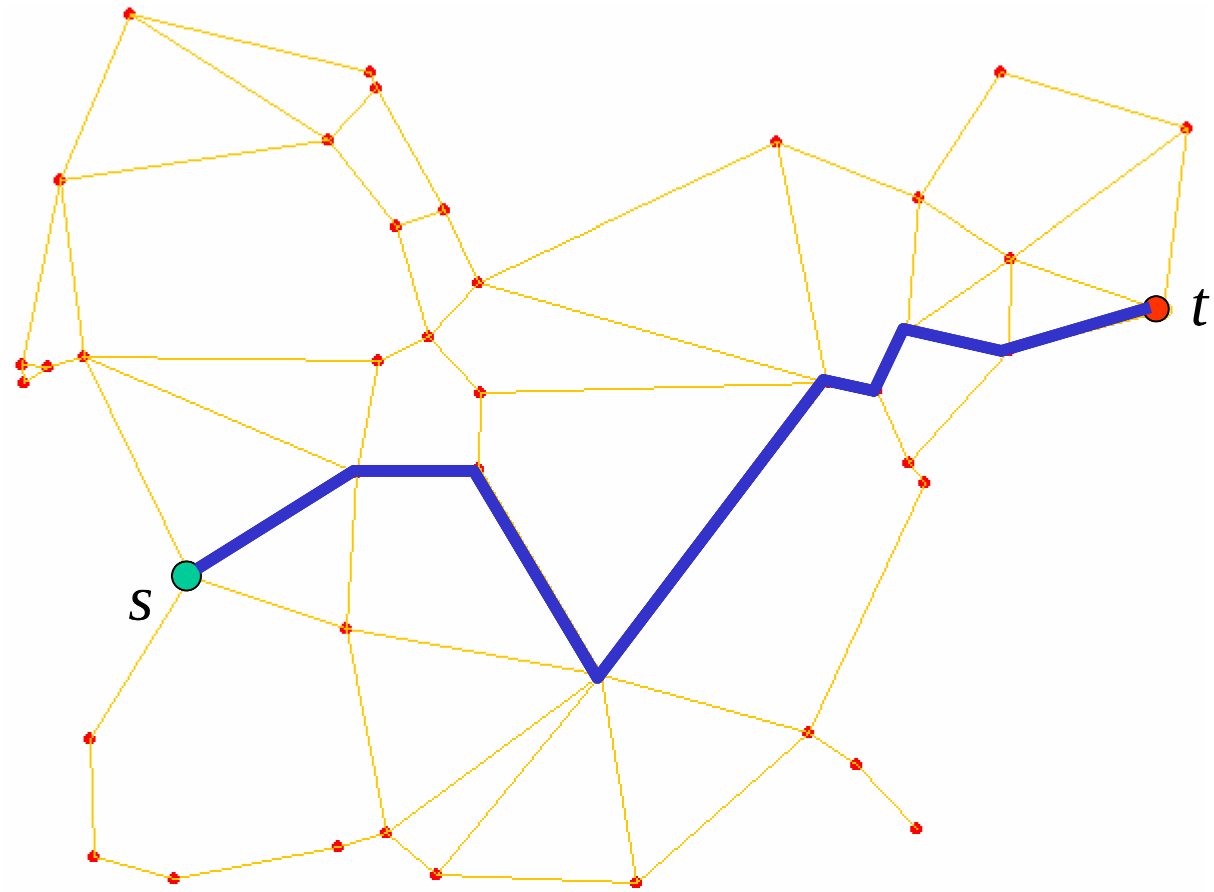


CR always moves a packet situated at the vertex *v* to the neighbor *u* of *v* that minimizes the angle $\alpha = \angle u,v,t$

Memoryless algorithm

Compass Routing

Kranakis, Urrutia 99

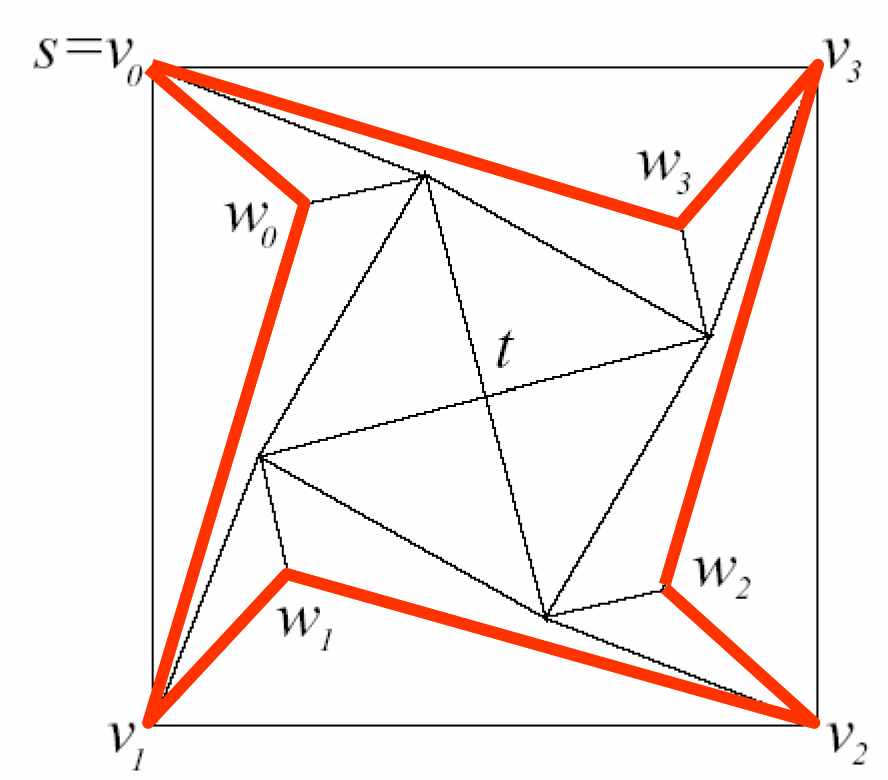


Memoryless algorithm

Compass Routing

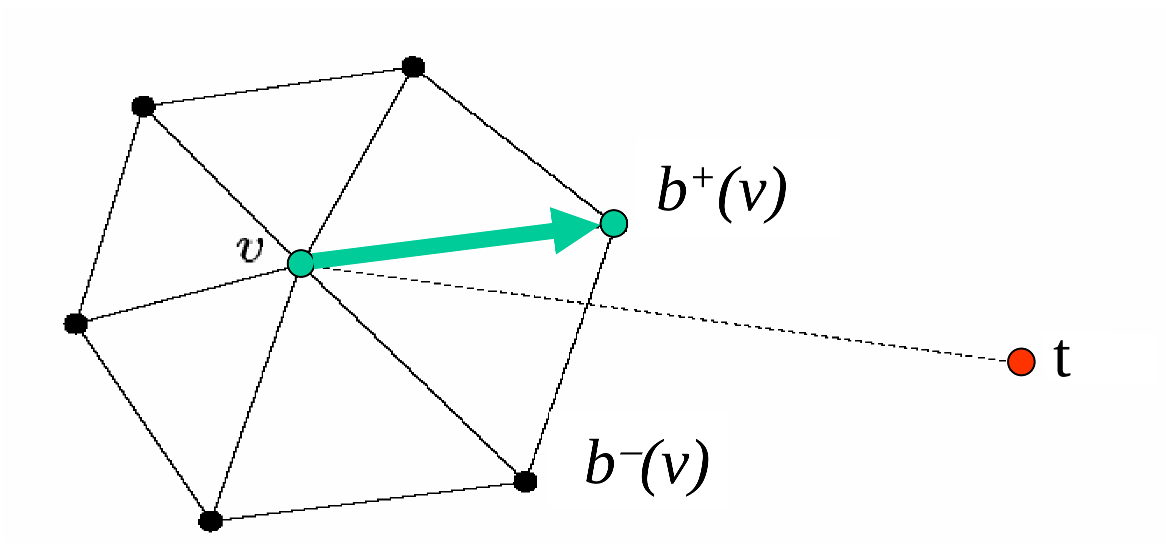
Kranakis, Urrutia 99

- Might fail, even in a triangulation.
  When we try to go from *s* to *t*, we travel around the cycle
  $v_{0,}w_0v_1w_1v_2w_2v_3w_3v_0w_0$ ....
  shown in red.
- Works well for Delaunay triangulations.



Memoryless algorithm

Greedy-Compass Routing Bose, Morin 99

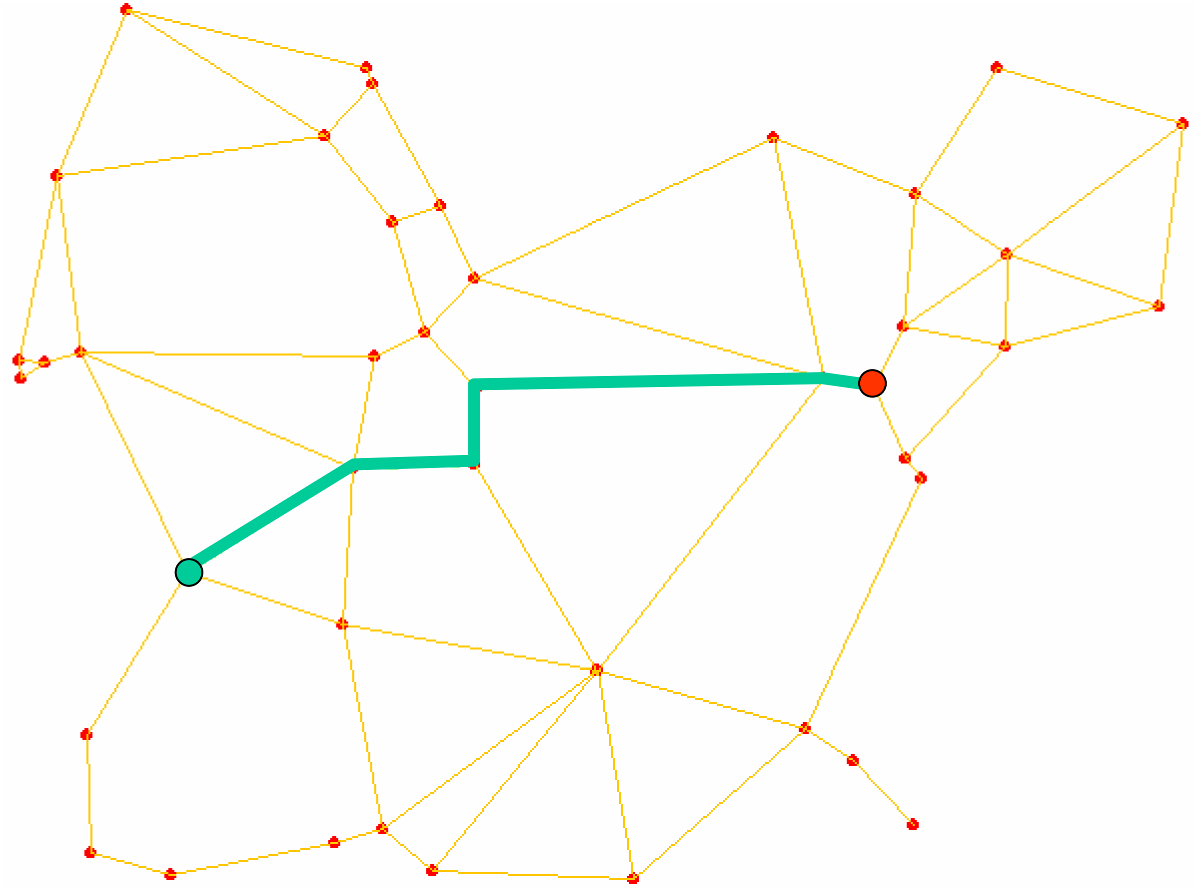


GCR always moves a packet situated at the vertex v to $u \in \{b^+(v), b^-(v)\}$ such that $dist(u, t)$ is minimized

Memoryless algorithm

Greedy-Compass Routing

Bose, Morin 99



Memoryless algorithm

Greedy-Compass Routing

Bose, Morin 99

Greedy-Compass Routing is a memoryless algorithm that is not defeated by any triangulation.

But, what happen for general graphs?

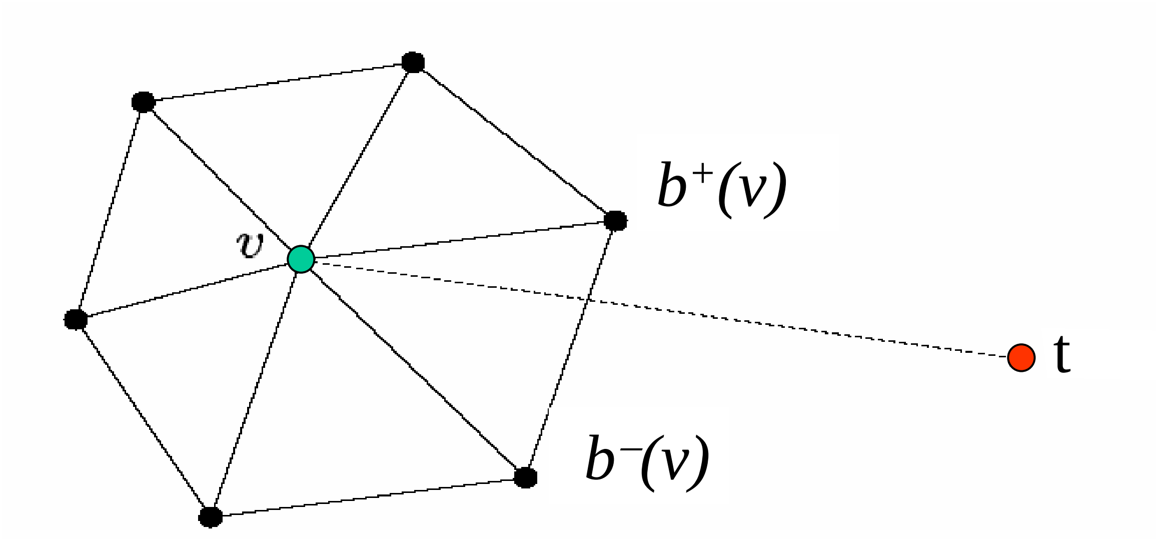
There is no deterministic memoryless routing algorithm that works for all convex subdivisions

Memoryless algorithm

Randomized

Random-Compass Routing

Bose, Morin 99



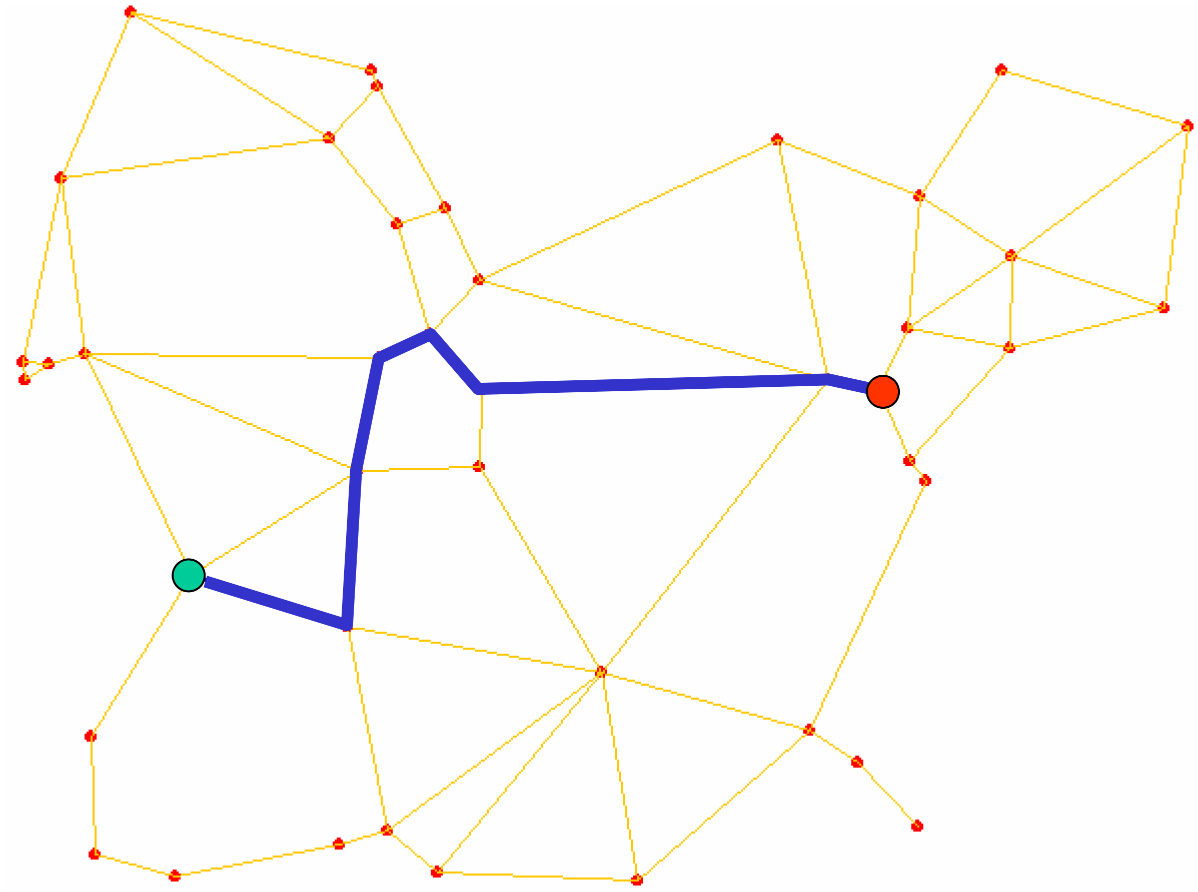
RCR moves a packet situated at the vertex v to one of $\{b^+(v), b^-(v)\}$ with equal probability

Memoryless algorithm

Randomized

Random-Compass Routing

Bose, Morin 99

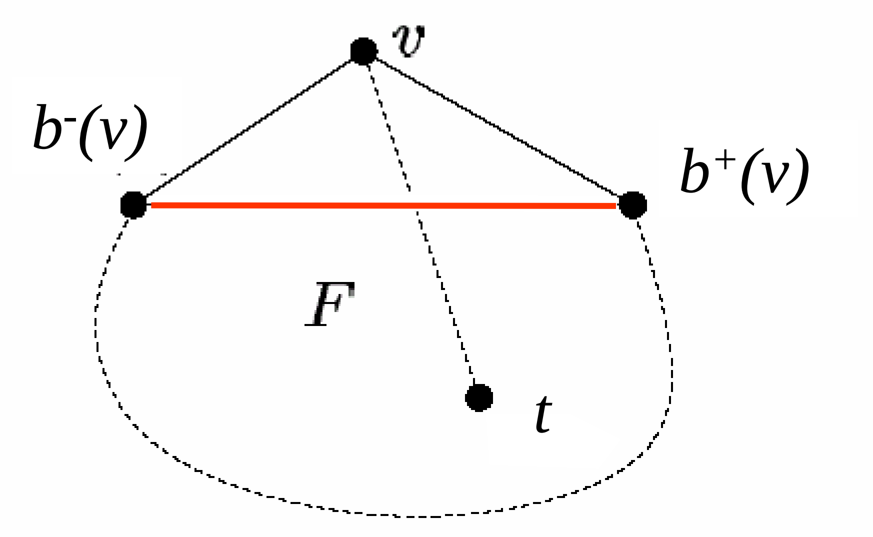


Memoryless algorithm *Randomized*

Random-Compass Routing

Bose, Morin 99

- Works for any triangulation
- Works for any convex subdivision



Memoryless algorithm

Randomized

Random-Compass Routing

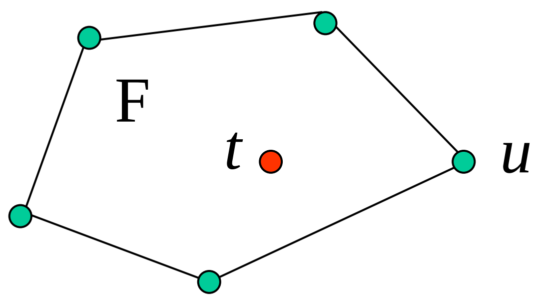
Bose, Morin 99

Works for any convex subdivision

Assume that there is a convex subdivision G with *s*, *t* such that the probability of reaching *s* from *t* using RCR is 0

Then there is a subgraph H, $s \in H$, $t \notin H$

with $b^{+}(v)$ and $b^{-}(v)$ en H for all $v \in H$



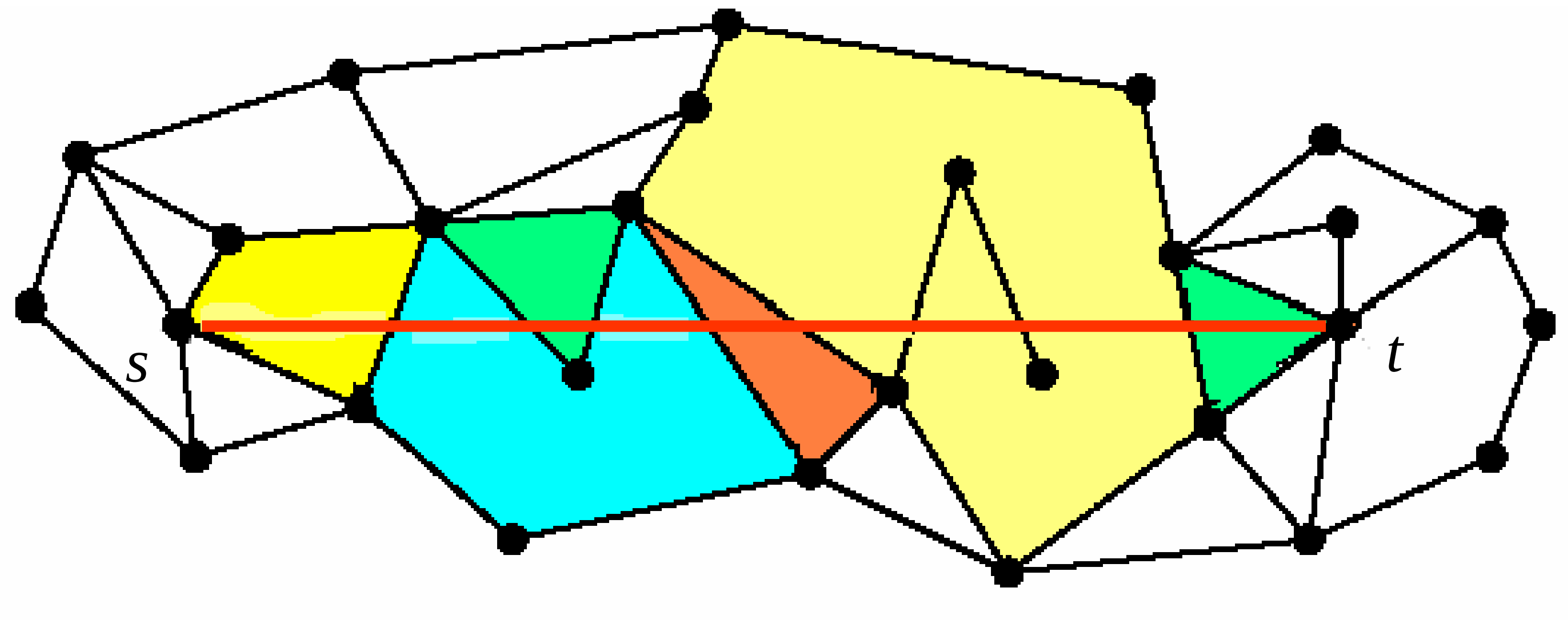
F convex face of *t*

Since G is connected, it must be *u* on the boundary of F, such that $b^{+}(v)$ or $b^{-}(v)$ is in the interior of F

O(1)-Memory algorithm

Face Routing

Kranakis, Urrutia 99



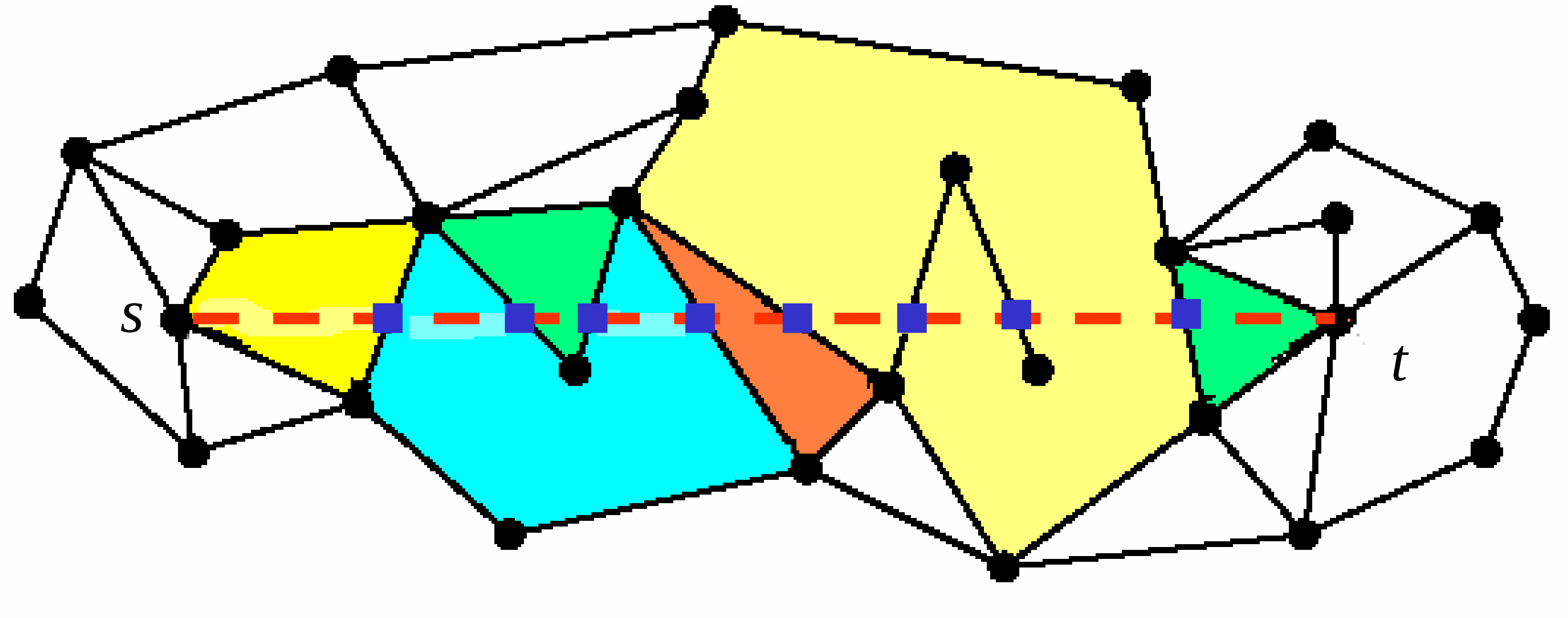
Route along the boundaries of the faces that lie on the source-target line

O(1)-Memory algorithm

Face Routing

Kranakis, Urrutia 99

1. Let F be the face incident to the source *s*, intersected by line *s*,*t*



2. Explore the boundary of F; remember the point *p* where the boundary intersects with (*s*,*t*) which is nearest to *t*.
Go back to *p*, switch the face and repeat 2 until you hit the target *t*

O(1)-Memory algorithm

Face Routing

Kranakis, Urrutia ‘99

Theorem: Face routing terminates on any simple planar graph in O(n) steps, where n is the number of the nodes in the network.

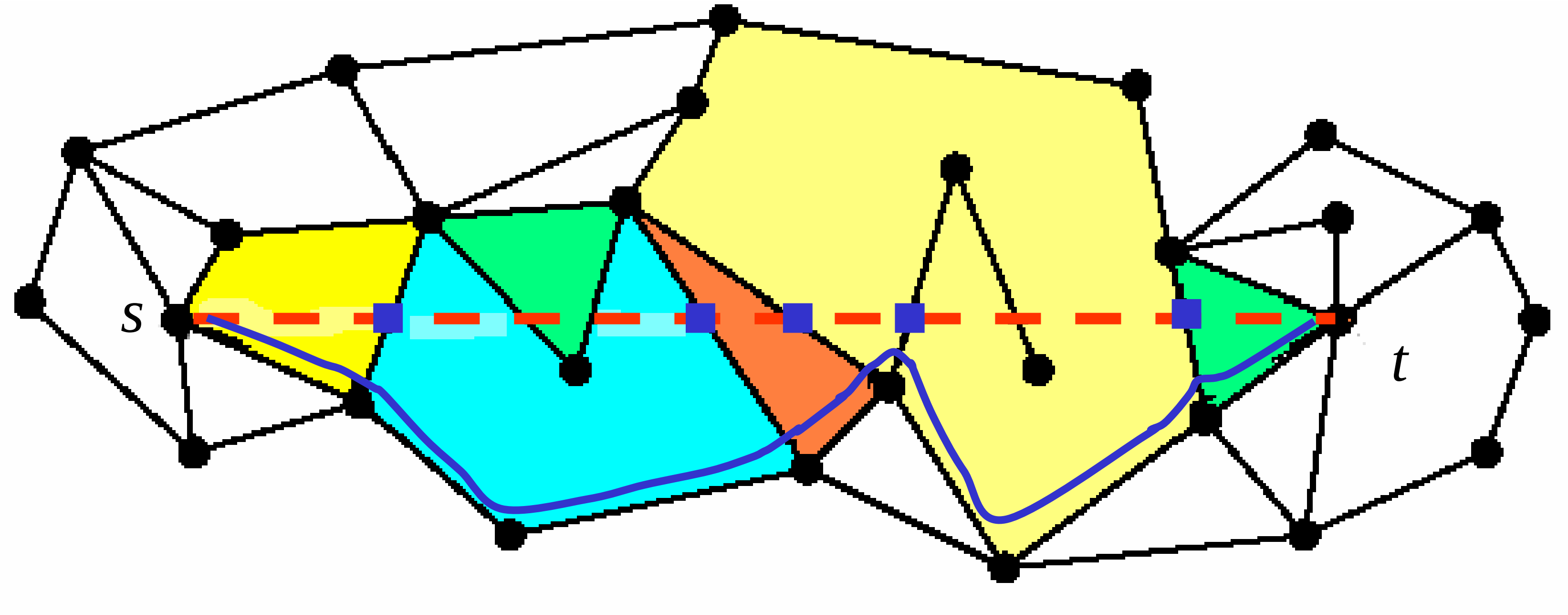
Proof: It is straightforward to see that we reach the the destination *t*
We can order the faces that intersect the (*s*,*t*) line, therefore
we never visit a face twice. Each edge is in at most two faces,
therefore each edge is visited at most 4 times.
Since a simple planar graph has at most 3n–6 edges, the
algorithm terminates in O(n) steps.

O(1)-Memory algorithm

Face Routing 2

Bose, Morin, Sojmenovic, Urrutia '99

Idea: Traverse F until reaching an edge that intersects *(s,t)* line at some point *p*, switch the face and repeat until to reach *t*

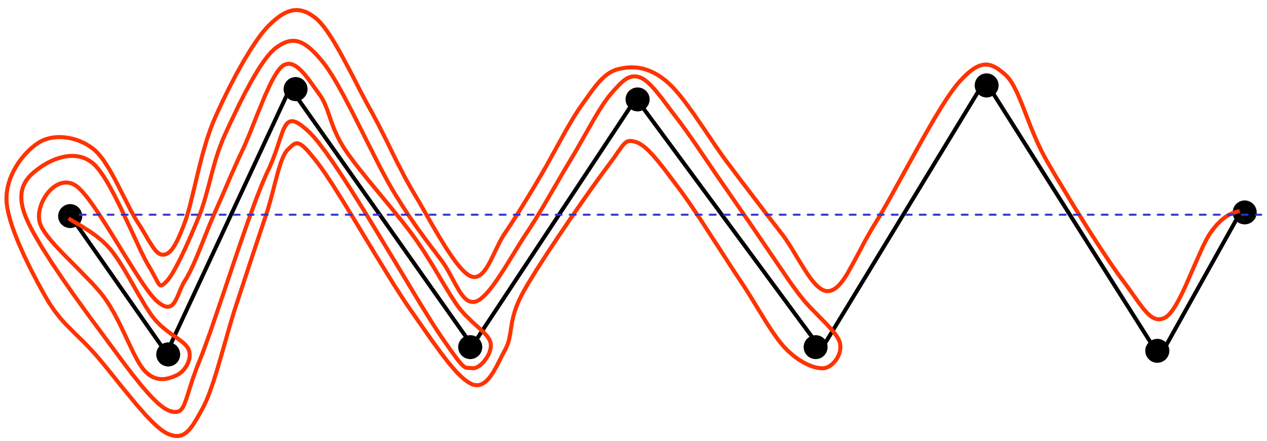


O(1)-Memory algorithm

Face Routing 2

Bose, Morin, Sojmenovic, Urrutia '99

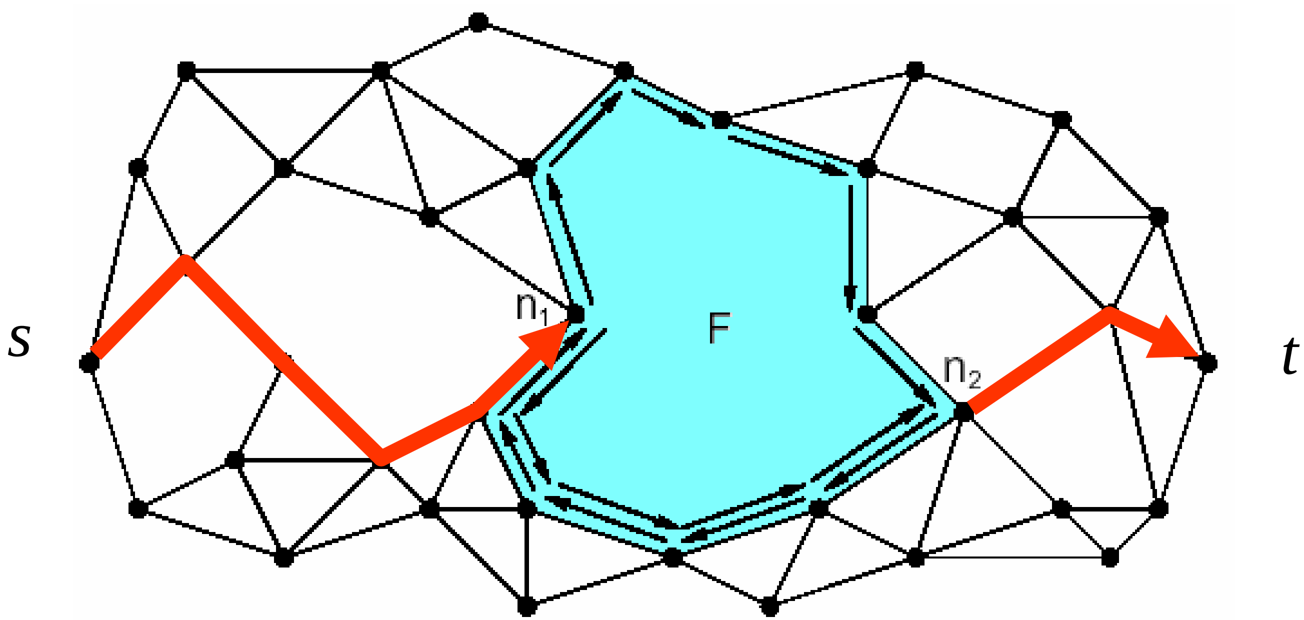
Efficiency: Face-2 reaches *t* in a finite number of steps, seems to be practicallymore efficient than face routing, but in pathological cases it may visit $\Omega(n^2)$ edges of G



Hybrid strategie

BMSU '99 KWZ '03

GOAFR - Greedy Other Adaptive Face Routing



1. Route greedily as long as possible
2. Circumvent "dead ends" by use of face routing
3. Then route greedily again

Competitiveness of paths (and routing algorithms)

Greedy Routing

Compass Routing

Random-Compass Routing

are "good"?

Now, we must consider the length of the path found by every routing algorithm

An algorithm **A** is *c*-competitive for G if $$\frac{A(s,t)}{SP(s,t)} \leq c$$

A(s,t) length of the path found by A

SP(s,t) length of the shortest path between *s* and *t*

Competitiveness of paths (and routing algorithms)

Greedy Routing

Compass Routing

Random-Compass Routing

are "good"?

Euclidean metric or Link metric?

> **Theorem**
> For any constant *c*, there exist Delaunay triangulations for which none of the GREEDY, COMPASS, GREEDY-COMPASS and RANDOM-COMPASS algorithms are *c*-competitive

Competitiveness of paths (and routing algorithms)

Greedy Routing

Compass Routing

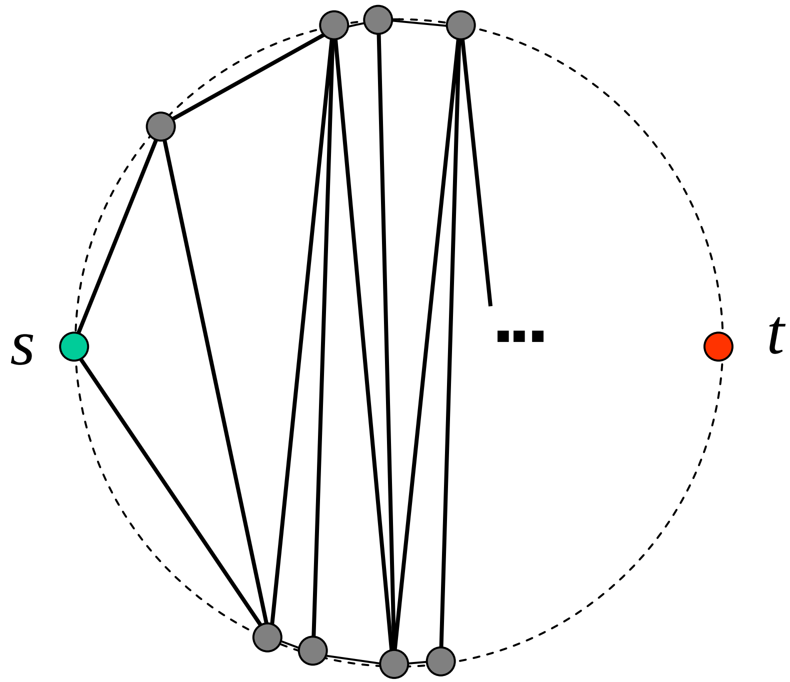
Randomized Compass Routing

are "good"?

Greedy routing

$SP(s,t) = (\pi/2)\, dist(s,t)$

$A(s,t) = O(n)\, dist(s,t)$



Is there exists any competitive algorithm?

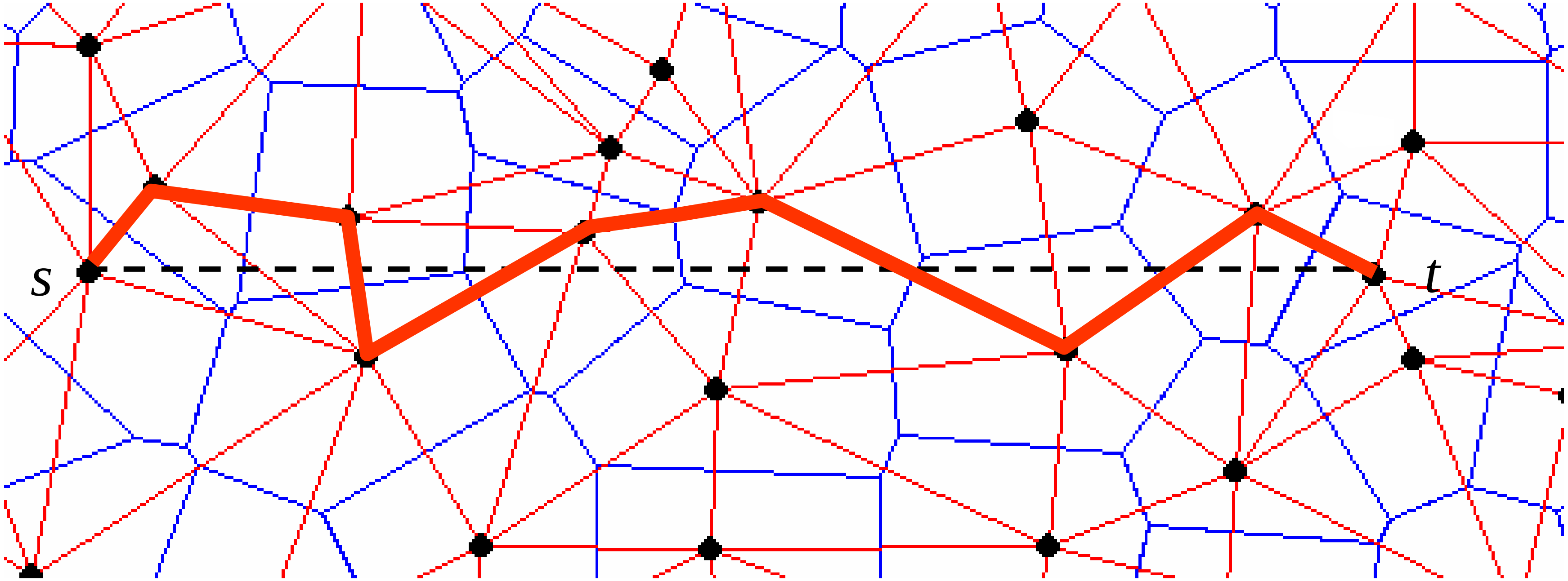
Bose, Morin ‘99
Parallelal Voronoi Routing is an O(1)-memory routing that is *c*-competitive for all Delaunay triangulations under the euclidean distance metric.

Bose, Morin , + ‘00
Under the link distance metric, no routing is *c*-competitive for all Delaunay triangulations.

And for all greedy or minimum weight triangulations

Competitive algorithms

Voronoi routing



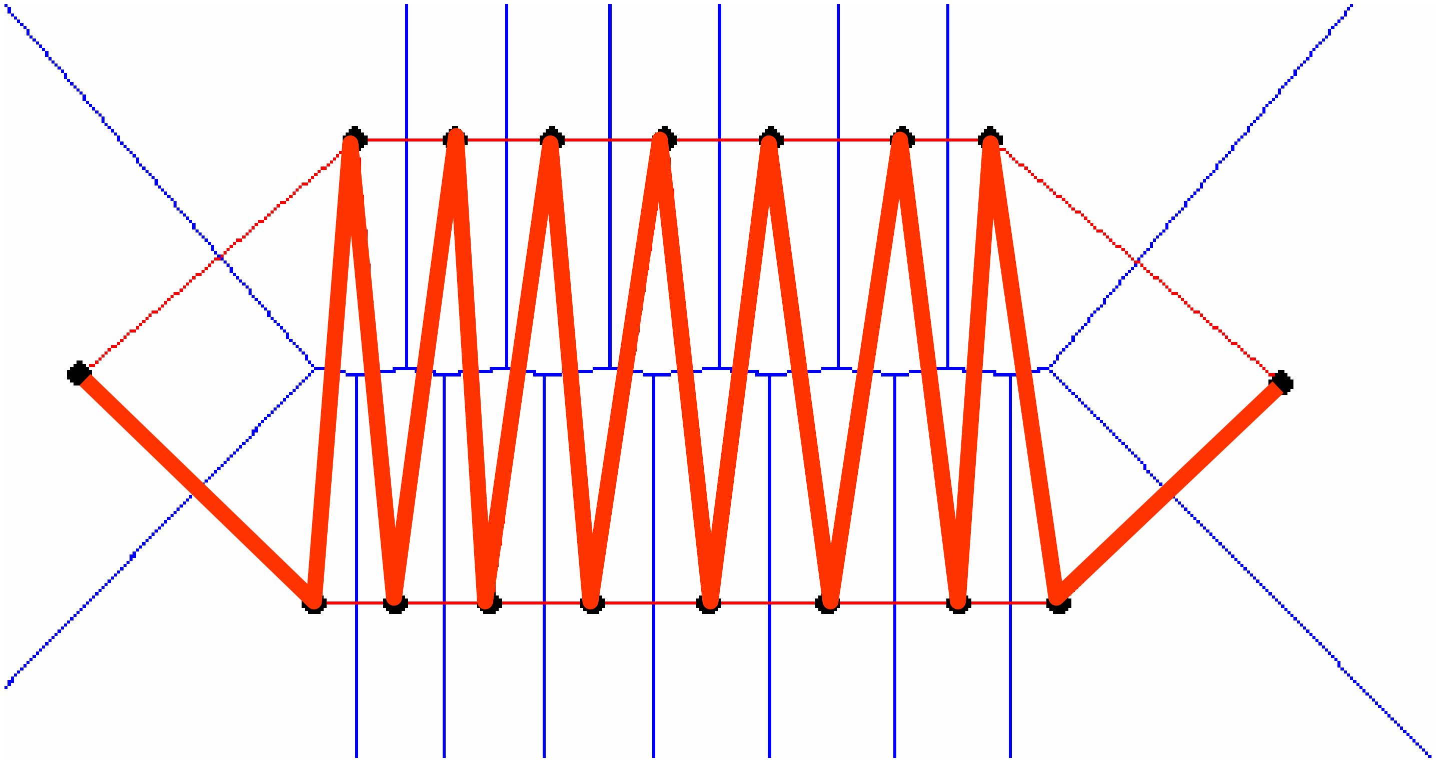
Segment *st* intersects the Voronoi regions $R_1, R_2, \ldots, R_m$ (in order)

VR moves the packet along the path P_V: $s = v_1, v_2, \ldots, v_m = t$
where v_i is the vertex defining R_i

Competitive algorithms

Voronoi routing

Voronoi routing is an O(1)-memory routing algorithm



VR is not *c*-competitive for all Delaunay triangulations

Competitive algorithms

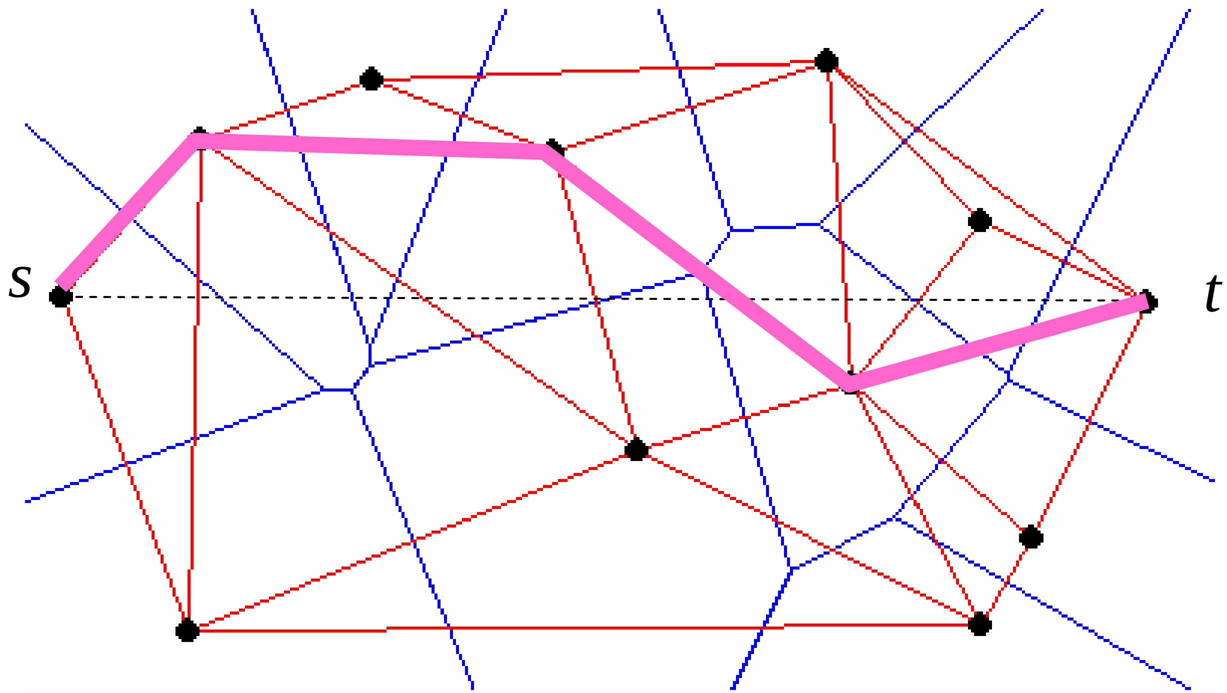
Parallelal Voronoi routing

Properties of DT (Dobkin et al. ‘90)

1. **DT** approximates the complete euclidean graph, i.e.

$$\mathrm{SPDT}(a,b) \le k \cdot \mathrm{dist}(a,b)$$

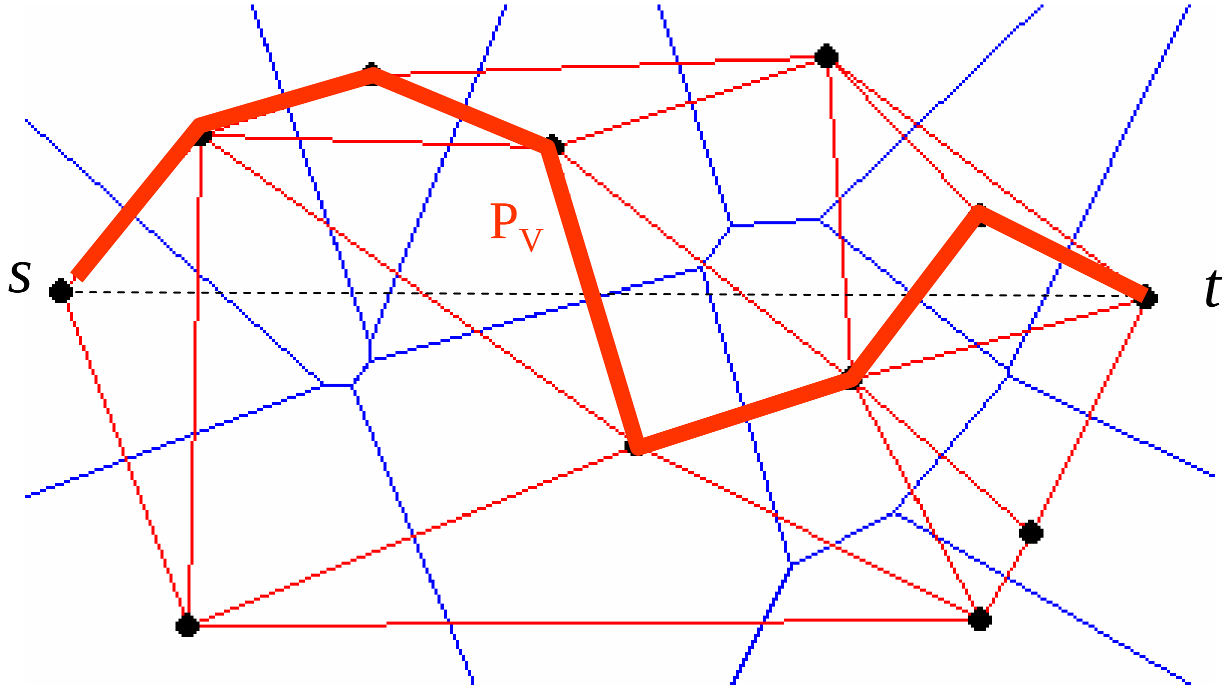
SPDT(a,b) length of the shortest path between a and b in DT



Competitive algorithms

Parallelal Voronoi routing

Assume that *s* and *t* both lie on the x-axis

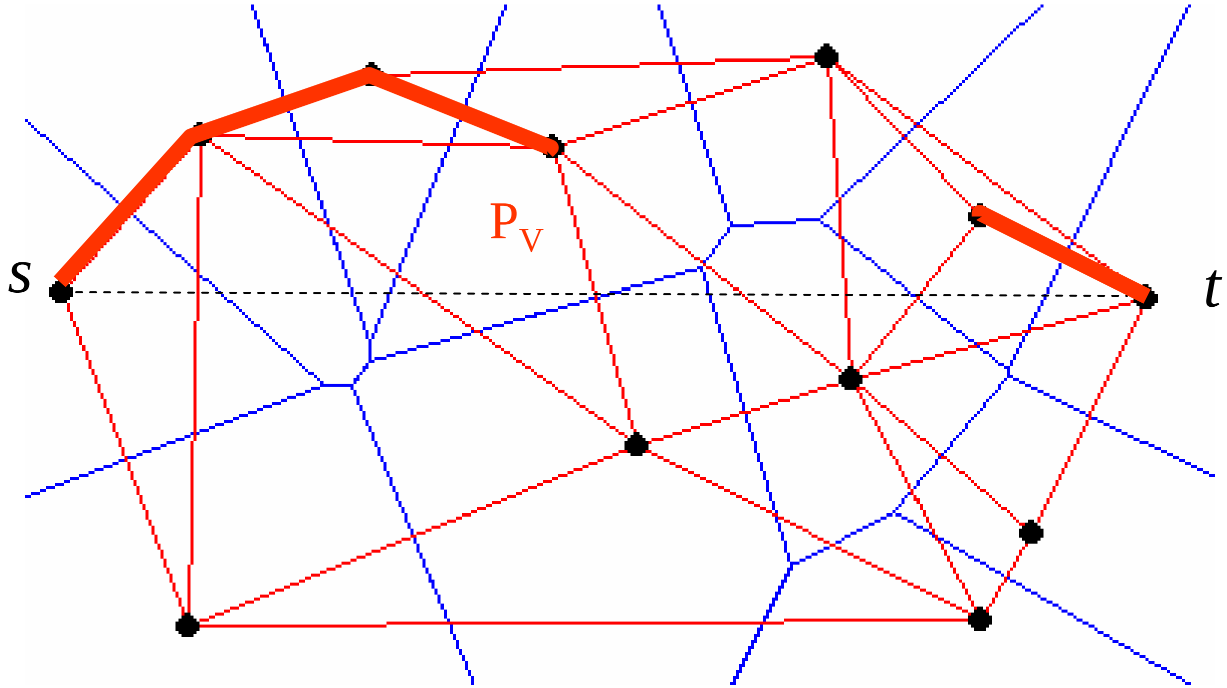


2. The path P_V is x-monotone
3. The length of the subpaths above the x-axis is $\leq (\pi/2)\ \text{dist}(s,t)$

Competitive algorithms

Parallelal Voronoi routing

Assume that *s* and *t* both lie on the x-axis



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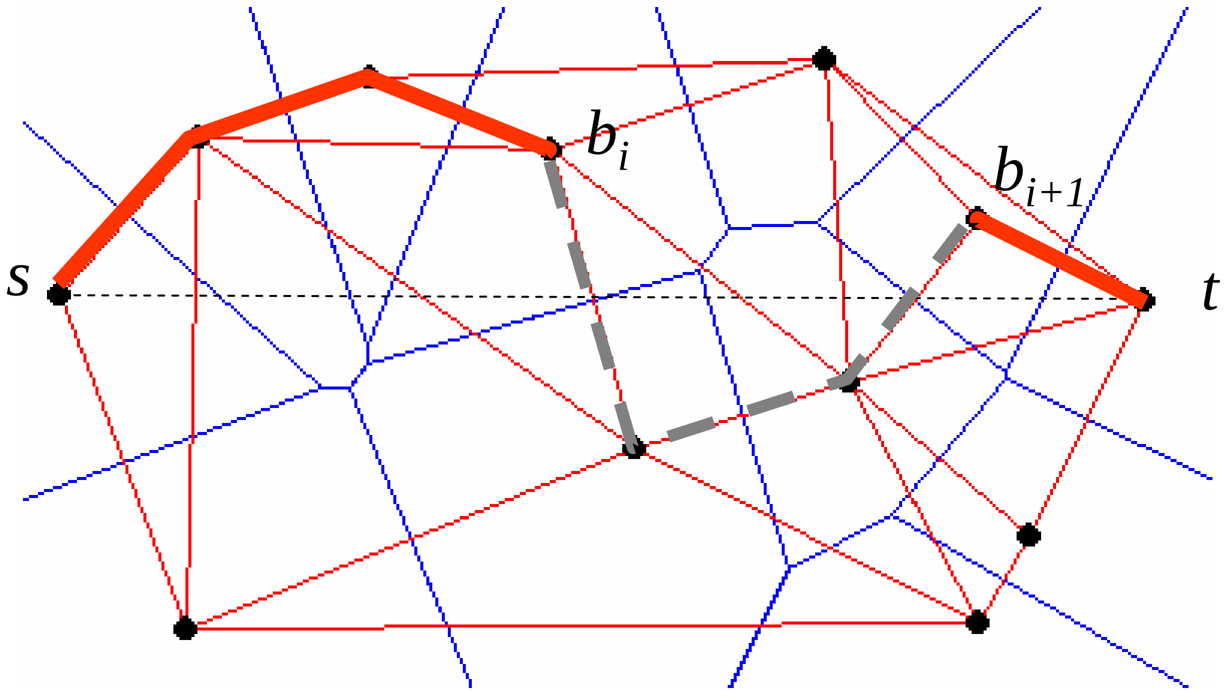
Competitive algorithms

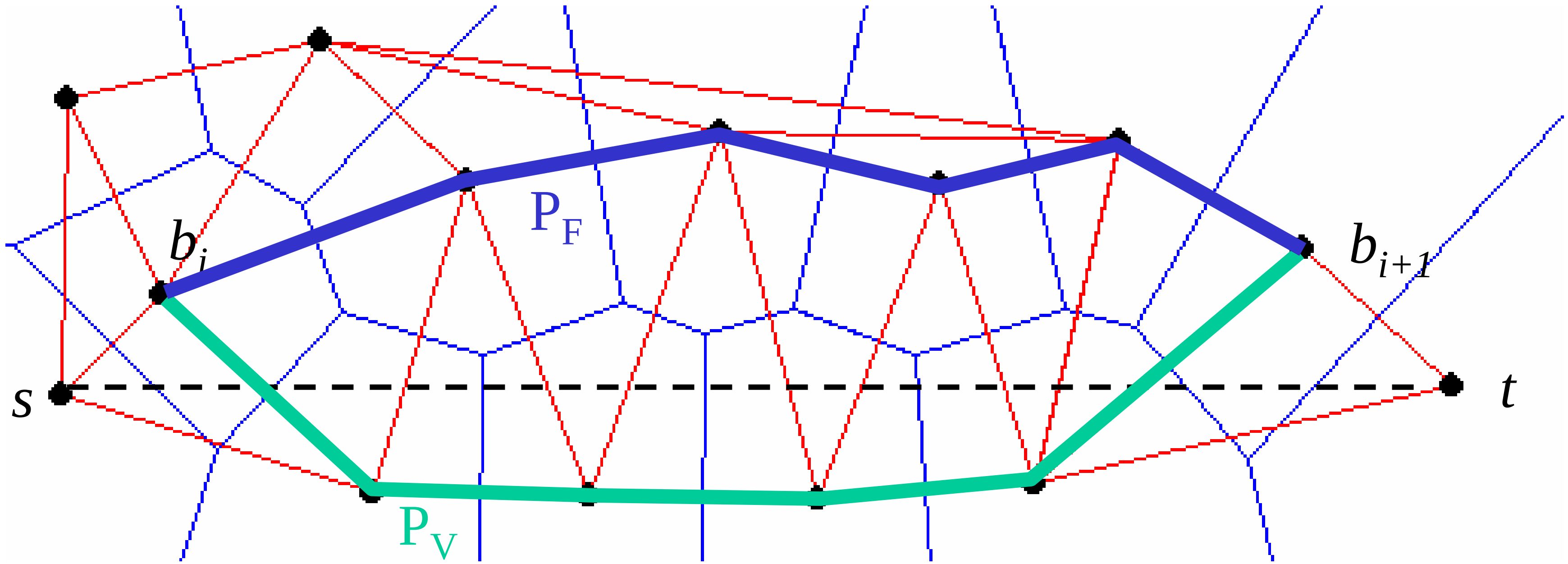
Parallelal Voronoi routing



What happens between two vertices b_i b_{i+1} of P_V situated above x-axis, when the Voronoi path is not a direct edge?

Competitive algorithms

Parallelal Voronoi routing



$|P_V| \leq c^* (x_{i+1} - x_i)$ or $|P_F| \leq c^* (x_{i+1} - x_i)$

PVR travels an overall distance of, at most

$$(9c^* + \pi / 2)\, dist(s,t)$$

Open problems

There is competitive routing algorithms for Delaunay, Greedy and Minimum Weight Triangulations (EUCLIDEAN METRIC)

1. For what other classes of geometric graphs do competitive routing algorithms exist?

There is no competitive routing algorithms for Delaunay, Greedy and Minimum Weight Triangulations (LINK METRIC)

2. Is there a class of geometric graph that admits a competitive routing algorithm ? (meshes don't count)

REFERENCES

- E. Kranakis, H. Singh, J. Urrutia, *Compass routing on geometric networks*, CCCG'99
- P. Bose, P. Morin, *Online routing in triangulations*, ISAAC'99
  SIAM Journal on Computing, 33(4):937-951, 2004.
- P. Morin, *Online routing in geometric graphs*, Ph. D. 2002
  http://cg.scs.carleton.ca/~morin/publications/
- Distributed Computing Group, ETH Zürich
  R. Wattenhofer. http://dcg.ethz.ch/index.html