

# Topological realization of algebraic structures

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In this talk we introduce basic notions of the theory of Alexandroff spaces. These topological spaces are the same as partially ordered sets from a categorical point of view and they have very interesting algebraic invariants (homology groups or homotopy groups).

Generally, for every topological space  $X$ , one can consider its group of automorphisms as an object of the topological category  $Top$ , denoted by  $Aut(X)$ , or the homotopical category  $HPol$ , denoted by  $\mathcal{E}(X)$ . Our main result establishes that for every homomorphism of groups  $f: G \rightarrow H$  there exists an Alexandroff space  $X$  satisfying that  $Aut(X) = G$ ,  $\mathcal{E}(X) = H$  and the natural homomorphism of groups between  $Aut(X)$  and  $\mathcal{E}(X)$  corresponds precisely to  $f$ . Additionally, we explore other realizations problems related to homology groups, homotopy groups and the actions of  $Aut(X)$  or  $\mathcal{E}(X)$  on them.

## References

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2. P.J. Chocano, M.A. Morón and F.R. Ruiz del Portal, On some topological realizations of groups and homomorphisms, *Trans. Amer. Math. Soc.*, 375(12), 8635-8649, 2022.