

Certain uniqueness theorems of generalized Markov numbers

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Abstract

The equation $x^2 + y^2 + z^2 = 3xyz$ is called the Markov equation and its positive integer solutions are called Markov triples. A positive integer b is called a Markov number if there exist positive integers a and c such that (a, b, c) is a Markov triple. In 1823, Frobenius conjectured the following: For every Markov number b , a Markov triple (a, b, c) with $\max\{a, b, c\} = b$ is unique. This is called the uniqueness conjecture. There are several partial affirmative results on this conjecture. For instance, the uniqueness conjecture is true for prime Markov numbers.

In this talk, we consider the equation

$$x^2 + y^2 + z^2 + k(xy + yz + zx) = (3 + 3k)xyz$$

for $k \geq 0$. This equation was introduced by Gyoda and Matsushita, and is called the k -generalized Markov equation. For this equation, k -generalized Markov triples, k -generalized Markov numbers, and the uniqueness conjecture are defined similarly. I will explain that the uniqueness conjecture is true for certain k -generalized Markov numbers.