Understanding the topological structure of attractors

Héctor Barge

Universidad Politécnica de Madrid

Abstract

In this talk we shall examine the problem of determining under what circumstances a metric compactum can be realized as an attractor of a dynamical system on a manifold. We shall see that there are two different ways of approaching the problem, depending on whether we consider compacta in the abstract or embedded in a prescribed "phase space".

After surveying some classical results by Günther and Segal for flows and Kato for homeomorphisms that solve completely the problem in the abstract case, we shall focus on the problem of determining those subcompacta of the 3-dimensional euclidean space $\mathbb{R}^3$ that can be realized as attractors for a dynamical system defined on $\mathbb{R}^3$. We shall present the complete solution of the problem for a family of compacta known as *toroidal sets*. These results have been obtained in collaboration with J.J. Sánchez-Gabites.