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Seminario de Investigación
Dpto. Matemática Aplicada
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11:30 Riordan Arrays and related topics¹

**13:00 Counting Combinatorial Statistics in Dyck
Paths and Polyominoes²**

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Riordan Arrays and Related Topics

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Abstract

Let $\kappa[[z]]$ be the ring of formal power series over an integral domain κ . A Riordan array generated by a pair of functions (g, f) in $\kappa[[z]]$ where $f(0) = 0$ is an infinite lower triangular matrix constructed out of the g and f in such a way that its k th column generating function is gf^k for $k \geq 0$. In many contexts, we see that the Riordan arrays are used as a machine to generate new approaches in combinatorics and its applications. This talk is devoted to introducing the notion of Riordan arrays and related topics.

Counting Combinatorial Statistics in Dyck Paths and Polyominoes

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Abstract

In this talk we discuss some combinatorial results of a family of Dyck paths called *non-decreasing*. A path is non-decreasing if the y -coordinates of its valleys form a non-decreasing sequence. This family has several interesting combinatorial properties that are related with other combinatorial objects. One of them, for example, is a bijection between the non-decreasing Dyck paths and the directed column-convex polyominoes. Using this bijection we study several new combinatorial statistics for these polyominoes. In this talk we show enumerative results and some statistics of several aspects of non-decreasing Dyck paths. Most of the results are given using Fibonacci numbers, generating functions (using the symbolic method), and Riordan arrays. This talk is based on a joint work with E. Czabarka, R. Flórez, and L. Junes.

Key words : Non-decreasing Dyck path, generating function, Fibonacci number, Riordan arrays.

References

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