ETSI Montes, Forestal y del Medio Natural Matemáticas II. Curso 2019-2020.

Hoja 4: Cálculo diferencial para funciones de varias variables

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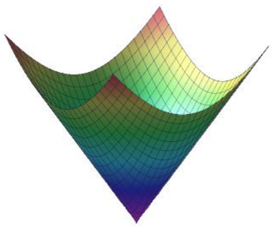
$\boxed{4.1}$ Representa las curvas de nivel correspondientes a $k = -2, -1, 0, 1, 2$ y describe cómo son para un valor arbitrario de k.

a) $z = sen(x+y)$ b) $z = 2x^2 + y^2$ c) $yz = 1$ d) $z = \dfrac{y}{x}$ e) $z = \dfrac{x+y}{x-y}$ f) $z = \dfrac{x+y}{x^2+y^2-1}$

$\boxed{4.2}$ Determina cuál es la gráfica de las funciones siguientes y cuál su conjunto de curvas de nivel:

$$f(x,y) = e^x \cos y \qquad g(x,y) = \tfrac{x-y}{1+x^2+y^2} \qquad h(x,y) = \operatorname{sen}(xy)$$
$$j(x,y) = 4\sin x \cos y \qquad l(x,y) = \sin(x+y) \qquad m(x,y) = \sqrt{x^2+y^2}$$

(a)


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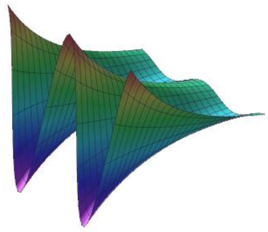

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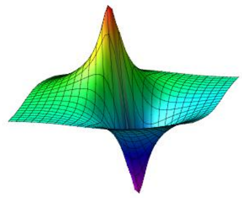

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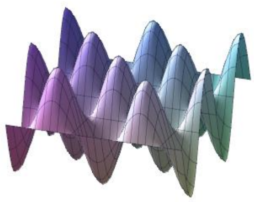

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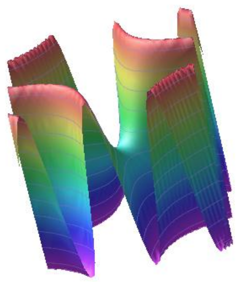

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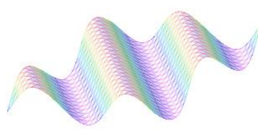

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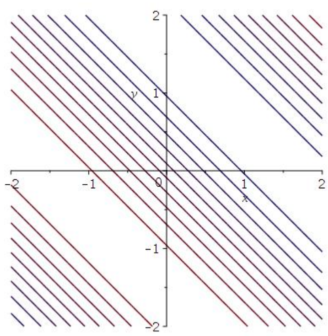

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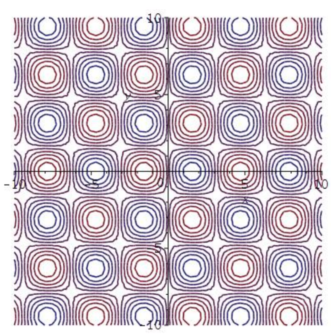

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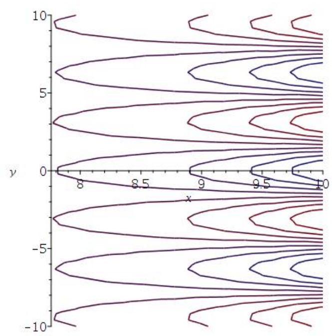

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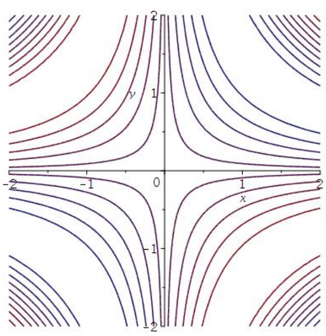

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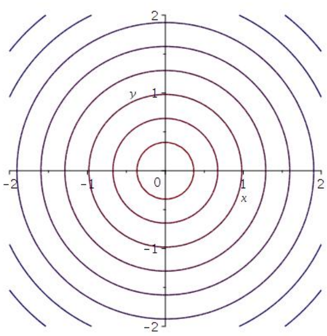

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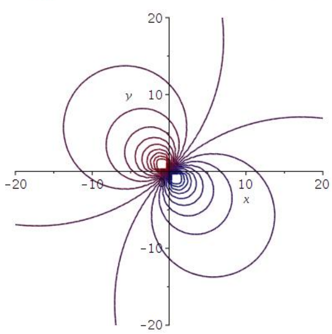
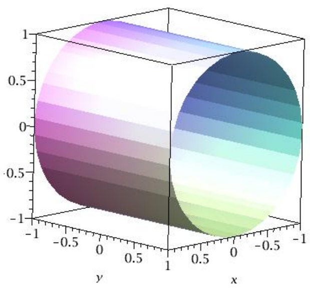
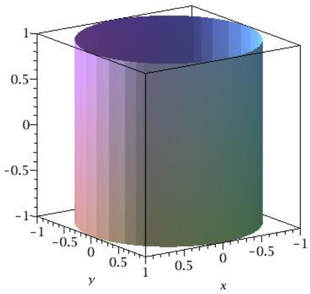
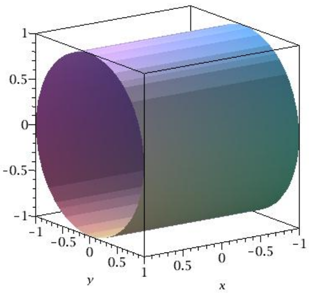


$\boxed{4.3}$ Determina qué superficie se corresponde con cada una de las ecuaciones

a) $x^2 + y^2 = 1$ b) $x^2 + z^2 = 1$ c) $y^2 + z^2 = 1$.

(1) 

(2) 

(3) 

$\boxed{4.4}$ Estudia cómo son los conjuntos de nivel de las siguientes funciones:

$$f(x,y) = 3x - 7y \quad g(x,y) = \frac{x}{x+y} \quad h(x,y) = x - y^2 \quad j(x,y) = \frac{x}{y} \quad k(x,y) = \frac{e^x}{y}$$

$\boxed{4.5}$ Estudia la existencia de límite en el origen de las funciones

$$f(x,y) = \frac{x^3 + y^5}{x^2 + y^4} \quad g(x,y) = \frac{xy^2}{x^2 + y^4} \quad h(x,y) = \frac{x|y|}{\sqrt{x^2 + y^2}}$$

$$j(x,y) = \frac{6xy}{x^2 + y^2 + 1} \quad k(x,y) = \frac{e^{xy} - 1}{x} \quad l(x,y) = \frac{x^2 + y^3}{x^2 + y^2}$$

$\boxed{4.6}$ Analiza la continuidad de las siguientes funciones:

$$f(x,y) = \begin{cases} \frac{x^2+2xy^2+y^2}{x^2+y^2}, & (x,y) \neq (0,0) \\ 0, & x = 0 = y \end{cases} \qquad g(x,y) = \begin{cases} \frac{\operatorname{sen} xy}{x^2+y^2}, & (x,y) \neq (0,0) \\ 0, & x = 0 = y \end{cases}$$

$$h(x,y) = \begin{cases} \frac{\operatorname{sen} xy}{x}, & (x,y) \neq (0,0) \\ 0, & x = 0 = y \end{cases} \qquad j(x,y) = \begin{cases} \frac{2xy}{x^2+y^2}, & (x,y) \neq (0,0) \\ 0, & x = 0 = y \end{cases}$$

$$k(x,y) = \begin{cases} \frac{x}{x+y}, & x + y \neq 0 \\ 1, & x + y = 0 \end{cases} \qquad l(x,y) = \begin{cases} (x^2+y^2)\cos\frac{1}{x+y}, & x + y \neq 0 \\ 0, & x + y = 0 \end{cases}$$